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JUNE 1954

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—PRESIDENT J. C. WARNER
Carnegie Institute of Technology

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School Science and Mathematics

We Cover the Earth

Is read by subscribers in every state of the Union, all provinces of Canada, and thirty-three foreign countries.
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SCHOOL SCIENCE AND MATHEMATICS

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EDITOR OF THE PROBLEM DEPARTMENT RETIRES
A NEW EDITOR TAKES OVER

In October 1931 Prof. G. H. Jamison's name first appeared at the head of our Problem Department. Now, after twenty-three years of most faithful service, Professor Jamison retires. But his retirement from teaching and editing just means a change in activity. He plans a term of travel for his own pleasure and education and a chance to write for the benefit of the thousands who have enjoyed his excellent department. Thanks a million and adieu as editor. God be wi ye. We shall be looking for your articles concerning our American neighbors to the south.

Dr. Margaret F. Willerding, Assistant Professor at Harris Teachers College and in charge of adult education at Washington University, St. Louis, will take over the work of the Problem Department at once. She has already been at work, getting fully acquainted with the details and planning the fall issues.

Professor Willerding's preparation for the work she is now starting is excellent. She received her A.B. from Harris Teachers College, then began teaching in 1942 in the Gardenville Schools, St. Louis, in charge of the 7th and 8th Grade science and mathematics. In 1946–47 she was a graduate fellow at St. Louis University and the following year was Instructor at Washington U. Since 1947 she has been in charge of adult education at both Washington U. and St. Louis U. She received her Ph.D. in mathematics and physics from St. Louis and since 1948 has been Assistant Professor of Mathematics at St. Louis U. Her publications include articles in the *Bulletin of the American Mathematical Society*, *The Mathematics Magazine*, *The Mathematics Teacher*, *School and Community*, and SCHOOL SCIENCE AND MATHEMATICS. She is also joint author with Bartoo and Osborn of *Algebra*

A Second Course, published by Webster Publishing Company in March 1954. She has been active in the mathematical groups of Missouri and is at present Chairman of the Missouri Affiliated Group of the National Council of Teachers of Mathematics. She has traveled extensively throughout the United States and Mexico.

Send all your problems and solutions for the Problem Department to Dr. Margaret F. Willerding, Harris Teachers College, 5351 Enright Avenue, St. Louis 12, Missouri.

PAUL E. KAMBLY EDITOR FOR BIOLOGY

A good man always has plenty to do. Professor John E. Potzger has received a number of promotions, being advanced to head the Botany Department at Butler University and he is now President of The Ecological Society of America. This requires all his time so he has had to give up our major editorial work in Biology. We have appreciated his work on this journal since January 1941 and wish him continued success.

To replace Professor Potzger we are pleased to announce that Prof. Paul E. Kambly of the University of Oregon will now take over. Professor Kambly was born at Cisna Park, Illinois, May 14, 1908. He graduated from the Watseka High School in 1925 and the Illinois State Normal University at Normal in 1930. His first teaching was done at Athens (Ill.) High School. In the fall of 1933 he moved to Iowa City, where he taught in the University School and took graduate work in the University, receiving his Master's degree in 1934 and the Ph.D. in 1939. He remained at Iowa until 1946 as a teacher of high school biology, science supervisor in the elementary school, and as teacher of the methods courses for both elementary and secondary teachers. In the fall of 1946 he became Professor of Education and Director of Supervised Teaching at the University of Oregon. Here he still has charge of the science methods courses. Professor Kambly has published extensively in many of the leading journals including the *American Journal of Botany*, *Proceedings of the Iowa Academy of Science*, *Science Education*, SCHOOL SCIENCE AND MATHEMATICS, and many others. He is also joint author, with Carroll Lane Fenton, of *Basic Biology* and workbooks and laboratory manuals in biology. Many no doubt remember his conservation series, *Your Land and My Land*, published by the National Wildlife Federation in 1942. If you want to go hunting or fishing, or if you want to take some excellent photographs of the wonderful wild scenery, the beautiful and productive valley of the Williamette, or the mighty forests of the Oregon coastal range, Dr. Kambly can take you around and give expert advice on the use of your camera.

SCIENCE IN THE KINDERGARTEN

ERMA KOTCHIAN
Lincoln School, Watertown, Wisconsin

Should science teaching begin in the kindergarten? Can science be taught in the kindergarten? These questions have been submitted by many teachers to the "experts." Their answers have been mixed and at times confusing. The best way to determine the answers to these questions is to use the scientific approach and try it. This is what I did at the Lincoln School in Watertown, Wisconsin.

The science experiences started with planning a bird feeding station. The children discussed and planned the desired type of feeding station. The next step was to get it constructed. For this the class prevailed upon a boy in an upper grade room. When the feeding station was finished, the problem of locating it had to be solved.

Two children from the class took yardsticks and accompanied the janitor outside. The janitor held the station in several positions. The best position was selected by the children in the classroom. The children wanted to be able to see the station while sitting in their chairs in the classroom. Jumping up from the chairs would frighten the birds away. The yardsticks were used to measure how high it had to be placed.

Now a post of the proper length was needed. Did anyone living on a farm have a fence post not in use? Steve, one of our boys, thought there might be one on their farm. The next day Steve and his mother brought in the post.

The feeding station was fastened to the post and the post placed in a hole in the ground. We could see the station without jumping out of our chairs, because it was just outside one of the classroom windows.

What do we feed birds? Many inquiries by the children brought information. Ground chick feed and suet are good. Sunflower seeds are good. Chick feed, suet, and sunflower seeds were brought in. In fact, the sunflower seeds were still in the flower head. The children enjoyed looking at and feeling of the part of the flower holding the seeds. The children were anxious to help pick the seeds out of the shells. During this process the children drew many designs similar to those they saw on the head of the sunflower and the shells covering the seeds. One of the children reminded us that we had saved the seeds from our pumpkin when we made our Jack-O-Lantern. These were also to be used for food.

Two children were selected as those responsible for placing fresh feed on the station each day. They were allowed to do this for a period of one week when two others assumed the job. This was and still is one of the favorite jobs in the kindergarten.

The first birds to come were the sparrows. The children were careful not to make quick or noisy moves until the sparrows became accustomed to them. The sparrows ate the food put out by the children. Finally, a chickadee came! The eventful day was marked on our calendar. Everyone was excited, even though the tiny bird stayed only a few seconds. The bird returned periodically, each time staying only a second or two, but someone always saw him.

I asked, "Were you sure it was a chickadee? How do you know?" The answers came from the children:

"The chickadee has a black cap."

"The sparrow's head is grayish-brown."

"The chickadee is a little smaller than the sparrow."

"The sparrow's bill is broad and strong for cracking seeds."

"The chickadee's bill is sharp for digging insects and insect eggs from the bark of trees."

"The chickadee eats the suet."

"The sparrow does not eat suet."

"They both eat seeds and ground grain."

After several visits, the chickadees became more accustomed to the children and returned more often and stayed for longer periods. The children loved them. One day it was snowing and one little chikadee perched on the roof of the feeding station. He lifted up nis head and caught a snowdrop in his bill.

The children were most interested in this act and suggested, "He is thirsty." The children assigned to feeding put on their wraps and went outside to place a small container of water on the feeding station. Fresh water was placed on the station each day after that.

One day the children excitedly exclaimed, "The water! It's frozen!" Everyone rushed to see and feel the ice. Each day there was the same amazement.

A thermometer was brought to school. "Do you know what this is?", I asked. One or two children knew it was a thermometer. The thermometer was returned to the room and we watched the line again. The line was short when cold and long when warm.

I explained that when the top of the red line is right here where it reads about three-0, then it is cold enought to freeze water. A string was tied to the thermometer and it was hung outside the window. Each day the children eagerly rushed to the window and reported often that the line was below three-0 (30°, an approximate reading), and the water would soon be frozen. Sometimes they reported that it was getting warmer outside because the line was above three-0.

Trouble was encountered with the thermometer. It was hung by a string and would be turned around by the wind, so that we could

only see the back. One girl suggested that we tape it in position, which we did. I asked the children, "What tipped the thermometer over?" Their response was, "The wind."

This learning experience with thermometers developed naturally through following this other interest in birds. Other uses for the thermometer, such as when to wear warm clothes, were discussed.

The most frequent visitors to our feeding station were sparrows and chickadees, but occasionally a downy woodpecker would pay us a visit.

The children noticed and looked for birds when away from school. They were constantly discussing birds they or some members of their family had seen and fed at home. At times they would come to school with an oral description of a bird and would attempt to identify it by looking at pictures in bird books.

Charts were made that included a picture of birds seen by the children. For example, one chart had this heading, "I Saw a Sparrow." On this chart was a picture of a sparrow. Under the picture were listed the names of all the children who had seen a sparrow. Similar charts were made for the chickadee, downy woodpecker, hairy woodpecker, pigeon, owl, bluejay, cardinal and others. These charts were thumbtacked to the bulletin board. As children reported seeing more birds, new charts were added to those already on the bulletin board. As more space was needed the first charts were removed and compiled into booklet form for continued reference. It was easy to compile this booklet because all charts were preplanned with 9×12 paper.

Children make errors in identification, but develop an interest in and a habit of looking at and for birds. Their ability to observe is sharpened. They give evidence of the ability to carry on critical thinking. Word identification, and hence, vocabulary is increased. Desirable attitudes are developed through pursuing this interest.

Through this activity, children also developed desirable social characteristics. The air of friendly cooperation was always noted. They accepted their responsibilities as privileges. This activity further led to other interests in numbers, words, and expression through pictures.

From my experience with this unit, I say science not only should and can be taught in the kindergarten, but it must be, if schools are to take advantage of children's interests and help children learn and grow in desirable directions.

Lunch box liner, made of a white, washable plastic, has two compartments with snug fitting lids for juicy foods and one compartment for solid foods. It fits standard-sized lunch pails.

THE FIVE MOST NEEDED RESEARCH INVESTIGATIONS IN THE TEACHING OF SCIENCE AND IN THE TEACHING OF MATHEMATICS*

GEORGE GREISEN MALLINSON
Western Michigan College of Education, Kalamazoo, Michigan

The participants in the Round Table on Research in Science and Mathematics were Kenneth E. Anderson, School of Education, University of Kansas; Kenneth E. Brown, Specialist in Mathematics, Office of Education; Foster Grossnickle, State Teachers College, Jersey City, New Jersey; Charles Myers, Educational Testing Service; Lynnette B. Plumlee, Educational Testing Service; Nathan S. Washton, School of Education, Queens College; J. Fred Weaver, School of Education, Boston University; and the chairman.

Prior to the Round Table several of the participants met and agreed that a general discussion would do little more than raise issues that had already been raised frequently at other meetings. Hence it was agreed that the time would be devoted to formulating the five most needed investigations in the teaching of each of the areas of science and mathematics. The participants, therefore, with the aid of suggestions sent by Howard Fehr of Teachers College, Columbia University, prepared a list of these investigations. The list follows together with some observations and explanations made by the participants.

MATHEMATICS INVESTIGATIONS

1. *Identification of the concepts and functional competencies in mathematics needed for the general education of all students at the secondary level.*

The participants were of the opinion that most of the studies in the area of objectives of the teaching of mathematics deal with skills of mathematics such as the ability to compute and to do square root rather than with concepts, competencies and understandings. It was believed further that major emphasis is needed on identifying and defining these concepts, competencies and understandings as they are suitable as objectives for general education.

2. *The background in mathematics needed for teaching courses in mathematics for general education.*

It was noted that recent surveys indicate that few if any colleges

* A report by the Chairman of the Round Table in Research in Science and Mathematics held at the Convention of the American Educational Research Association in Atlantic City, New Jersey on Monday, February 15, 1954.

offer courses designed to aid teachers in teaching the courses in mathematics for general education.

3. *Tests that present situations in which the methodology of mathematics is tested rather than computational skills.*

By methodology the Round Table meant concepts, competencies, and understandings. If these are considered as being desirable objectives, then tests should be designed to measure them.

4. *Techniques for teaching mathematics inductively and for teaching students to think mathematically.*

The participants agreed that while it is not difficult to state the objectives that are desirable, methods of their implementation are often lacking.

5. *Non-technical publications that summarize the implications and practical applications of research in the teaching of mathematics.*

Most classroom teachers and some professional edcuators have little opportunity to analyze the more or less technical literature that contains the research investigations in science and mathematics. Hence the Round Table strongly urges that the AERA prepare "laymens' reviews" in the areas mentioned such as the recent one prepared for the field of reading.

Science Investigations

1. *Methodology for teaching effectively attitudes and problem-solving skills of science.*

In general, it was agreed that the major concepts, understandings, attitudes and skills considered to be important objectives of science teaching have been identified, at least tentatively. However, there is little understanding as to how to teach effectively the latter two of these objectives, namely scientific attitudes and problem-solving skills (elements of scientific method).

2. *The background in science needed for teaching courses in science for general education.*

This was considered to be a major problem in the training of science teachers since science courses usually offered in college are departmental rather than "broad field." Yet most of the students who take science courses in high school take "broad field" courses.

3. *Tests that present situations in which the methodology of science is tested rather than scientific facts.*

This point is a corollary of Point 3 under MATHEMATICS INVESTIGATIONS.

4. *Techniques for teaching science inductively and for teaching students to think scientifically.*

This point is a corollary of Point 4 under MATHEMATICS INVESTIGATIONS.

5. *Non-technical publications that summarize the implications and practical applications of research in the teaching of science.*

The problem with respect to the availability of the implications and practical applications of research studies applies to science as well as to mathematics. Hence the Round Table strongly urges that the AERA prepare publications of this type in science as well as in mathematics. It was noted that the National Association for Research in Science Teaching had already suggested that such a publication be contemplated by the AERA.

HARVARD SUMMER COURSE

A new course on "Current Developments in Science" for secondary-school science teachers will be offered by the Harvard Summer School this year. A number of special fellowships are available for the course, which continues the series in Science Education initiated by former President James B. Conant of Harvard.

The course will last from July 6 to August 18 and will carry 8 semester-hours' credit. It will be taught by Professor Eugene G. Rochow, member of the Harvard Chemistry Department and pioneer in silicone chemistry.

The course is designed to help teachers (1) expand and strengthen their knowledge of basic concepts in the physical sciences, (2) interpret new developments in these fields, and (3) relate these developments and concepts to their own teaching. Current and recent work in chemistry, biochemistry, and physics will be discussed, and field trips will be made to nearby industrial and university laboratories. Among the topics to be discussed will be recent developments in blood chemistry and nuclear chemistry. In order to make the course as valuable to teachers as possible, ways in which some of these topics can be used in high-school classes will be demonstrated.

Some of the fellowships will cover registration and tuition fees for the course; others will include the fees plus allowances toward room, board, and travel expenses. Awards will be based on qualifications, need, and geographical location of applicants.

The course can be counted for credit toward the Ed.M. or the A.M. in Teaching by those who are admitted to one of these degree programs in the Harvard Graduate School of Education.

Also of interest to teachers will be a number of courses in the sciences, including astronomy, biology, chemistry, physics, and mathematics, and a course on "The Philosophy of Science" by Professor Max Black of Cornell University.

Further information about any of these courses and about the fellowships can be obtained from the Harvard Summer School, 2-N Weld Hall, Cambridge 38, Massachusetts.

SOME REFLECTIONS ON THE GENERAL MATHEMATICS SITUATION*

JOHN J. KINSELLA
School of Education, New York University, New York 3, N. Y.

If general mathematics means the mathematics which every person should learn no matter what his future vocation may be, then it might seem that there should be almost complete agreement on the content of general mathematics. The mathematics taught from the kindergarten through the eighth grade is evidently "mathematics which every person should learn" for it is required of every student. There is certainly "almost complete agreement" on this part of the general mathematics program, just as there is in the case of elementary algebra and plane geometry. However, beyond grade 8, especially at the ninth grade level, there is far from complete agreement. The content, sequence, and distribution of emphasis in courses of study and textbooks of general mathematics give a picture of wide variations in the planning of the courses. (1). Why is it that a situation almost resembling anarchy reigns in the "mathematics which every person should learn" beyond the eighth grade level?

A partial answer may be that the general mathematics movement is a relatively recent one in the history of mathematics education while arithmetic, elementary algebra, and plane geometry courses have reached a comfortable middle age. The history of general mathematics in this country began about the turn of the century. In those days only a small percentage of young people attended high schools or academies. Hence, it could hardly be expected that the democratic slogan of "education to meet the needs of all the children of all the people" would be shouted as loudly as it is today when nearly all boys and girls of the ninth grade age group come to school. Instead, the principal concern of the leaders in the teaching of mathematics in the early years of this century was a better organization of the mathematics curriculum. There was considerable interest in correlating arithmetic, algebra, and geometry and in getting away from extreme compartmentalization. The growth of mathematics, like the growth of science, was beginning to make specialization necessary. As it became more and more difficlut to treat mathematics as a whole, it became more and more necessary to deal with part of it at a time. This was no perfect solution, for while one new part was being learned by itself the parts which had been taught previously were being forgotten. Then it was observed that many of the important applications of mathematics called for the use of many parts of

* This article was contributed at the special request of the Editor.—G.W.W.

mathematics in concert. It seemed logical to these pioneers in the field of general mathematics that the basic and significant parts of arithmetic, algebra, and geometry could be taught simultaneously, or in parallel, by relating or correlating these parts.

In 1923 appeared the famous report, "The Reorganization of Mathematics in Secondary Education" (2). The authors of this document conceived of general mathematics as a means of giving the student a "broad view of the whole field of elementary mathematics," an opportunity to test his interest and ability in mathematics, and a glimpse of the use of mathematics in later vocational life. Referring to the junior high school years, these men recommended that "the course for these three years should be planned as a unit with the purpose of giving each pupil the most valuable mathematical training he is capable of receiving in those years, with little reference to courses which he may, or nay not, take in succeeding years" (3). They also said that "the mathematics of the early high school years must be considered worthwhile by the students on the basis of what it does for him here and now. His experience in other school subjects and life out of school should begin to mean more to him because he is taking a course in secondary mathematics" (4). They expressed doubts about the value of adult, consumer mathematics for young students. They wondered whether these boys and girls, in terms of interest and maturity, were ready for such items as stocks, bonds, and taxation.

It is easy to see that some of the questions which plague those concerned with general mathematics today may have had their origins in the first quarter of this century. Consider these issues:

1. Should general mathematics be a survey of arithmetic, algebra, geometry and simple trigonometry?
2. How should the "survey" be planned to allow for scope without superficiality and dilution, and for variety without scattering and loss of organization and pattern?
3. How can a program of general mathematics be made valuable and interesting to students "here and now" while preparing them to meet their "life adjustment" needs as future consumers, citizens, and home-makers?

Between 1923 and 1940 the secondary school population multiplied. The variations in the interests, purposes, and abilities of the students also increased. General mathematics as a happy combination of arithmetic, algebra, geometry and simple trigonometry seemed no longer sufficient for a large part of the new group. The distinguished authors of the Report of the Joint Commission of the National Council of Teachers of Mathematics and the Mathematical Association of America found it necessary in 1940 to recommend something new for the general mathematics of the ninth year. "Regardless, however, of how the pupil will some day earn his living, he will always be a

citizen, and a consumer of goods and services. Hence, there should be an earnest effort to include in the instruction in arithmetic many problems based on activities and interests of the ordinary citizen. Compared with practice in the past there should be much more work involving such topics as home-owning, mortgages, taxes, installment buying, insurance, investments, automobile expenses, debts, risks, health, food, budgets, building and loan associations, cooperative enterprises, and the like (5). This group was careful to add: "The Commission recognizes that the immaturity of the pupil will prohibit an exhaustive study of the topics suggested, any of which might be considered further in grade 12. . . . " (6). Like the 1923 group these men were a little worried about the value of adult, consumer mathematics for the adolescent group, but consoled themselves with the statement that "the information that the pupil receives may be more important than the benefit to be derived from the computations" (7).

The Joint Commission made very clear its belief in segregation of ninth grade students into more than one track when the size of the school made the practice administratively feasible. "Such data as these make it difficult to defend the practice of a single program in mathematics for all high school pupils, appropriate differentiated curricula seem to be called for" (8). It dismissed the argument that such segregation was undemocratic. It pointed to evidence that the "greatest good to the greatest number" resulted, at least in the learning of mathematics, when those who learned rapidly and those who learned slowly were assigned to separate classes. For the ninth year it presented two courses in general mathematics for the average and better student, one general mathematics course for the slower group, and a course in elementary algebra. These courses differed principally in the amount of attention given to algebra and consumer-citizen mathematics, and in the teaching methods recommended. The Commission did not demand that all of these courses be offered in one school.

This Joint Commission Report is undoubtedly the most comprehensive one available as far as the problem of general mathematics is concerned. The fact that three different courses in general mathematics were given as illustrations seems to indicate that the "mathematics which every person should learn" is not fixed but must vary with the ability of the students. The only other deduction that can be made is that if general mathematics is relatively fixed, then the slower learners must take more mathematics after the ninth year. However, the Report did not suggest any general mathematics for any later year except the twelfth.

At that time a semester course in social-economic mathematics

would be offered primarily for those in commercial courses and for those who expected to study the social sciences later. "Many of the topics are to a certain extent repetitions of similar work in the ninth grade, but it is expected that, because of the lapse of time, the pupil will need a review. Moreover, on account of their greater maturity, pupils will be able to grasp more advanced concepts and also do more thorough work" (9). Some of the topics suggested were measurement and approximate computation, review of fundamental operations, the simpler ideas of statistical methods, index numbers, household budgets, installment buying, investments, insurance, taxation, and topics involving national policy, such as crop control, social security, foreign exchange and the distribution of the national income.

A comparison of the 1923 and the 1940 reports reveals some interesting differences:

1. The 1923 Report would have all students follow the same program in the ninth grade; the 1940 Report would segregate students into different courses in terms of previous preparation and ability.
2. The 1923 Report placed much less emphasis on the mathematics needed by the consumer, home-maker, and citizen than did the 1940 Report.
3. The 1923 Report expressed the idea that the ninth grade would be the last time a considerable portion of the students would contact mathematics and, therefore, proposed courses that would reveal the scope of mathematics and give the student the mathematics most likely to be useful to him in the business and industrial world. The 1940 Report recommended that no year be considered a terminal year, and in its general mathematics proposals placed little stress on vocational needs but great emphasis on consumer-citizen needs.

In the same year as the Joint Commission Report the Progressive Education Association produced "Mathematics in General Education" (10). The proposals in this document were radically different from any that had appeared before. First, the purposes of general education were taken to be the development of individuals who could meet their *present* needs in four basic aspects of living, namely, personal, personal-social, economic, and vocational. In the process of meeting these four kinds of needs the students were to acquire those personal characteristics essential for democratic living. General mathematics would be the mathematics that would best serve the purposes of general education, as defined. The writers of this report decided that mathematics could make its major contribution in the development of persons who could solve their problems reflectively. The major mathematical concepts most closely related to this kind of problem solving were taken to be problem formulation, data, function, approximation, operation, symbolism, and proof. These were selected because they permeated much of mathematics, were likely to recur frequently in broad problems having mathematical elements, and could be generalized beyond the field of mathematics. To say the

least, mathematics was to be given a subsidiary role; the personal needs of the student and participation in democratic living were to be the major concerns.

This philosophy required a methodology that resembles the core approach of Harold Alberty (11). As a group of students attacked a problem related to meeting adolescent needs in one or more of the basic aspects of living, listed above, the need for mathematical background might become apparent. At that time, or a little later, the needed mathematics would be taught. After several of these mathematical experiences arising out of group needs had been obtained, some or all members of the group might meet for a period of time to extend, deepen, and organize mathematical ideas for their own sake. The teacher as guide would, at appropriate times, focus attention on one or more of the seven concepts essential to problem-solving. On other occasions, he would encourage students to extend these concepts to non-mathematical situations.

The only well-known attempt to put some of this doctrine into operation in the field of mathematics was described by Harold P. Fawcett in "The Nature of Proof" (12). However, the core approach was not used. Instead, such notions as undefined terms, definitions, assumptions, inferences, indirect proof, the need for testing converses, necessary and sufficient conditions, and truth vs. validity were applied not only to the field of geometry but to thinking in non-mathematical situations as well. By means of these procedures the method of proof used in mathematics was extended and made functional in the general education of the students.

It is difficult to assess the impact of "Mathematics in General Education" on the general mathematics of today, for the monumental difficulties of implementing its proposals, even if accepted, prevented any thoroughgoing description of any serious application of them. The word, "needs," with all of its various connotations does appear more frequently. The Joint Commission's concern that the needs of consumers and citizens be met through programs of general mathematics seems in harmony with meeting the "economic needs" of "Mathematics in General Education." General mathematics as generalized mathematics illustrated by Fawcett's work does not seem to conflict with the proposals of the other reports. The implementation of this concept of general mathematics must lean heavily, of course, on the transfer of learning and superior teaching. On the other hand, the effectiveness of any learning outcome acquired in the classroom for later functioning in any situation depends also on the transfer of learning. For instance, the use of consumer problems in the ninth grade general mathematics course carries with it the hope that the student will in the near and distant future be able to bring

his learning to bear in a different situation. The 1923 Report's concern that the mathematics taught must be of evident value to students here and now is in tune with the Progressive Education Association's emphasis on immediate vs. deferred needs. The latter group's emphasis on the importance of problem solving would have met, few objections in 1923 or 1940; certainly today those who plan programs of general mathematics would approve, for, if problem-solving ability is not developed, the chances of applying general mathematics to situtions which vary from those in which the initial learning took place are very "slim."

In 1945 appeared the very important "Second Report of the Commission on Post-War Plans of the National Council of Teachers of Mathematics" (13). Included in this Report was a list of 28 competencies which every future citizen should achieve if he were to be "functionally competent" and "mathematically literate." "General mathematics for the ninth grade is here defined as a course that includes and emphasizes the elements of functional competence as outlined in the Check List . . . " (14). This list was based on an analysis of the mathematics used in the activities involved in the basic military training of World War II inductees (15), and the mathematical needs of the citizen as a consumer. A final report of the Commission included a twenty-ninth item dealing with the ability to reason from hypothesis to conclusion in mathematical and non-mathematical situations. Except for the last item nearly the same objectives had been listed in the Joint Commission's general mathematics course for the slow learner.

This Second Report must also be classified as a very important one when its effects are considered. Textbooks and courses of study appearing since the publication of the Report indicate their close attention to the list of competencies as well as to the carefully stated "theses" of the document. The latter more or less provided a platform for the teacher of mathematics from the kindergarten through the junior college and teacher training institutions. It was suggested that no year should be considered as the terminal year for any student in mathematics. The need for general mathematics courses beyond the ninth grade was pointed out. The necessity for incorporating the ingredients of general mathematics, as defined, into the academic courses was forcefully stated. The Commission hoped that nearly all students would acquire these competencies by the end of grade 9. Subsequent investigations have shown that at least another year will be necessary in many cases to reach the desired goal. In a study of forty-three schools in Iowa Ohlsen found serious deficiencies in the attainment of the twenty-eight competencies through grade 12, even in the case of those who had been successful in algebra and

geometry (16). In forty-two schools in Nebraska Beckman found that at the end of the ninth grade, in both algebra and general mathematics classes, only about 50% mastery of the competencies had resulted (17). Beckman concluded that either (a) "our present curriculum in mathematics is poorly planned and developed, or (b) that the teaching of mathematics is poorly done in our schools, or (c) that the competencies as proposed by the Post-War Commission have not been accepted as valid objectives by those planning the curriculum of the schools" (18).

The present status of courses in general mathematics was investigated by Irwin in 35 states and 92 schools. "Seventy-four of the 92 schools offered either a three-track or a multiple-track mathematics program; sixteen offered a double-track program, either at the ninth and/or twelfth grade levels; twenty-nine offered related mathematics in connection with industrial arts and agricultural curricula; and nine of the schools offered special-interest mathematics courses for girls in home-making, household arts, or pre-nursing curricula" (19). She summarized the content of these general, non-traditional courses as follows: "Although several of the non-traditional mathematics courses offered were of the 'fused' or 'unified' type and many were of the 'consumer mathematics' type, the majority were courses in arithmetic of varying degrees of difficulty, stressing understanding of number relationships and the basic principles of mathematics together with the uses or applications of such knowledge." (20). She also reported that several mathematics education specialists of national reputation, who formerly agreed with the Second Report's recommendation of only two tracks, algebra for some and general mathematics for the rest, now felt that at least three tracks were necessary, that business arithmetic and consumer mathematics should be delayed to the upper high school level, and that a program for the "maintenance of the fundamental skills" or an "increased ability to use mathematical principles," to whatever extent possible, was needed badly.

This should be the place for "summary and conclusions." However, the problem is unsolved; the battle is still in progress. One thing is certain: *general mathematics has become special mathematics.* At least two or three kinds of general mathematics seem to be necessary to meet the varying abilities, needs and purposes of the ninth grade population. Two or three tracks starting at grade 9 will probably be required because of different destinations and different rates of travel. General mathematics, like general education, is no longer one menu for all; there is now more than one diet to follow in order to maintain good health.

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20. *Ibid.*

DR. RUSCH CHARTS THE CANCER PROCESS

A new map for scientists probing the unknown regions of cancer was laid down April 11 by one of the nation's leading cancer experts.

In his presidential address to the annual meeting of the American Society for Cancer Research, Dr. Harold Rusch charted the cancer process as part of a unified cancer theory he presented.

Dr. Rusch, who is director of the McArdle Memorial Laboratory, University of Wisconsin Medical School, said cancer is the result of an out-of-step destruction of a cell function—a change which is passed on to the cell's offspring—permitting unchecked growth.

The scientist, in presenting his concept of the cancer process, drew on research carried on in the field of cancer and in allied fields. He paid tribute to geneticists, biologists, biochemists, cytologists, and many other groups of scientists for the part they play in expanding the knowledge of cancer.

At the same time Dr. Rusch appealed for increased research along several lines pointed up by his review of past and current investigations and the concept which he developed from these.

MAGNETISM

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CHAPTER IV

PRESENT DAY CONCEPTIONS OF THE ELEMENTARY MAGNET

We saw in Chapter I that by breaking a magnet into two parts two dipoles were formed, and the question was raised whether subdivision could be carried on until finally partition did not produce a dipole. That final particle we agreed to call the magneton or elementary magnet. This chapter will try to show the evidence we have at present for such an elementary magnet.

Ampere's Discoveries. Once more let it be said that Ampere, over 125 years ago, made very clear the fact that magnetism was a secondary effect of electric charges being set into motion. In order to account for the phenomena found in a magnet, he had to postulate the flow of electricity in the molecules composing the magnet. This was a bold and brilliant stroke on his part, a concept to become productive of many new ideas about magnetism.

It must be remembered that Ampere knew nothing about electrons, protons, and the other members of the particle family. Apparently he simply conceived of electricity as a flow in the molecules of some subtle fluid about which there was little or no knowledge in his day. We, from our greater present day knowledge, would raise the question: do electric charges really percolate through the molecules and atoms, or are there charged particles spinning on their own axes—or, as Lodge[1] expressed it, "Each atom, whether in the steel or in the air, is the seat of a whirl of electricity, more or less faced round so as on the average to have its plane at right angles to the lines of force." Have we any evidence bearing upon the question of percolating or whirling electric charges?

The quotation from Lodge shows that for over 50 years physicists have been speculating on this problem. Today we think we have a very reasonable answer, one that seems to unify all of the magnetic phenomena with which we are acquainted.

Rowland's Experiment. Oersted's discovery showed that electricity flowing in a conductor produced a magnetic field, an effect which Ampere clarified and indicated that no magnetic fields could exist without electricity in motion. There still lingered, however, the feeling that perhaps electricity flowing in a wire, or even whirling in an atom, might be different from a static charge which was mechanically moved through space.

[1] Lodge, "Modern Views of Electricity," pp. 171, 90, 131, 314. 1889.

Maxwell, in his Treatise on Electricity and Magnetism, in 1873, was in doubt concerning this question: Is a piece of matter which is charged electrostatically and moving with high speed equivalent to an electric current in a wire? The first to mention the probable existence of such an effect was Faraday,[2] who said, "If a ball be electrified positively in the middle of a room and then be moved in any direction, effects will be produced as if a current in the same direction had existed." In order to show that no matter where or how the electricity was moved, a magnetic field was still produced, Rowland[3] carried out his famous experiment "On the Electromagnetic Effect of Convection Currents," and showed that Maxwell's theory could be answered affirmatively.

In Rowland's final form of apparatus, two parallel non-conducting disks were mounted on horizontal axes so they could be rotated simultaneously. These disks were gold plated, and when charged and rotated, the electric charges on the gold coatings were carried around in circles about the axes of rotation, so that one had in effect a double coil. When a sensitive magnetometer was placed at a point midway between the two disks and on the axis of rotation, there was in effect a double coil galvanometer for indicating the effect sought. Rowland found that reversing the direction of rotation reversed the direction of deflection of the magnetometer, and that with the same direction of rotation a change in sign of the electric charge used also reversed the direction of deflection of the magnetometer.

Since Rowland's day many other types of experiments have only confirmed the results found by him. For instance, the deflection of a stream of electrons in our modern television sets indicates that around the stream of flying electric charges there is set up a magnetic field. All this goes to prove that under any condition whatever, if an electric charge moves it sets up around it a circular magnetic field, as Oersted found in his original discovery.

Magnetism a Process of Something Turning. Ampere conceived of the magnetism of a permanent magnet as due to a circular flow of electricity within a molecule, and that in a magnetic field these magnetic whirls were aligned more or less in a common direction. Thus early in our development of a theory of magnetism, it was rather fundamental to assume that when a piece of steel was magnetized, as by stroking in Fig. 13, or by placing the steel in a very strong magnetic field, something turned around within the steel, and the resultant effect of the turning of all of these elementary magnets in a direction more or less parallel to the applied field, was the magnetized

[2] Faraday, "Experimental Researches," Vol. 1, Art. 1644.

[3] Rowland, *Phil. Mag.*, Vol. 27, p. 445, 1889. See also Pender, *Phil. Mag.*, Vol. 2, p. 179, 1901; Vol. 5, p. 34, 1903; Vol. 6, p. 442, 1903.

bar of steel as a permanent magnet.

This process of bringing a group of elementary magnets into the presence of a magnetic field, thus orienting them so that they align themselves more or less in the same direction, is called induction.

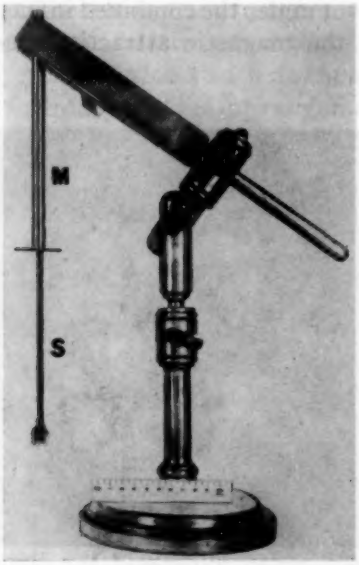

FIG. 45. A piece of soft iron *S* is induced by the magnetic field of the magnet *M* to attract iron filings to it.

The same thing occurs within the group of elementary magnets, regardless of the process by which they are turned. It may be by the process shown in Fig. 12, or by placing the piece of steel in a strong field such as that of a permanent magnet or of a solenoid, or by rotation, to be described later. Fig. 45 shows how a piece of soft iron, *S*, may be induced to pick up iron filings when brought into the presence of a magnetic field. The inductive process goes on in spite of the fact that between the permanent magnet *M* and the piece of soft iron *S* a piece of cardboard has been inserted. It would be just a bit more effective if the cardboard were removed and "contact" made as in Fig. 13.

As has been indicated, the idea that there were a great many elementary magnets within a body, such as a piece of steel, and that in the process of making a magnet (induction) the elementary magnets were all turned in the same direction and under their own mutual effect on each other they remained in that condition, continued for many years as the directing thought for further research.

Among those pursuing this line of reasoning, perhaps no one used

it more effectively or more productively in developing our theories of magnetism than did Ewing.[4] His book, *Magnetic Induction in Iron and Other Metals* (3d Ed.), is a classic among magneticians. Ewing thought of a piece of iron as an assemblage of innumerable "tiny magnets, each endowed with a fixed and constant magnetic moment, and wheeling about a pivot under the combined influence of the impressed magnetic field and the magnetic attractions and repulsions of its neighbors." (Darrow).[5]

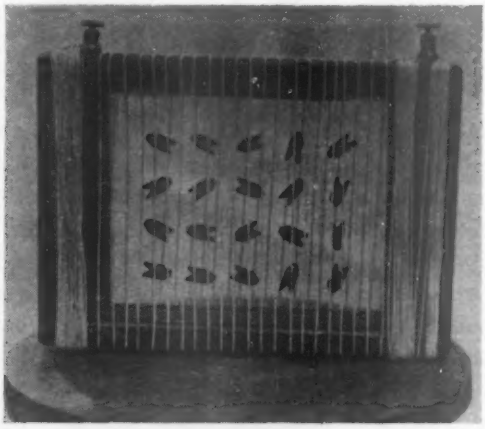

FIG. 46. Small pivoted magnets to represent the elementary magnets in a piece of steel.

Using small magnets mounted on pivot points, something like those shown in Fig. 46, Ewing studied groups of these "elementary magnets" to see how they would behave. With a group of 130 little magnets he found a surprising fidelity in their behavior to that of a bar of iron and its magnetization. This similarity will be amplified in discussing the actual process of magnetization (induction). Bozorth[6] has elaborated Ewing's model, and has been able to demonstrate with it the domain theory of magnetization. In succeeding chapters this will also be discussed.

While these models of Ewing's were helpful, the identity of the elementary magnet was still unknown. Was it something belonging to a molecule, as Ampere suggested, or to an atom? Was it something revolving like a planet in its orbit (à la Bohr) or was it an electrical whirl (Lodge)? Such questions really began to make themselves felt in the first quarter of the present century. Physicists began to direct

[4] Pp. 347–354, 3d. Ed. 1900. See later developments by Ewing, *Proc. Roy. Soc.* Edinburgh, Vol. 42, p. 97, 1922.

[5] Darrow, *Bell System Tech. Journ.*, Vol. 6, pp. 295–366, April 1927.

[6] Bozoroth, *The Amer. Jour. Phys.*, Vol. 10, pp. 73–78, 1942.

their efforts toward the solution of some of them, and made some crucial experiments.

The Comptons' Experiments. In sequence of unfolding the problems (not chronological), the two experiments by the two Comptons[7] and their colleagues are pioneer studies. Recalling that Ampere was following the idea that the elementary magnet was the molecule, Compton (K. T.) and Trousdale reasoned that "If the elementary magnet consists of a group of atoms such as the chemical molecule, the rotation of this elementary magnet into alignment with an applied external field will cause a displacement of the individual atoms. It is known, however, that the position of the spots on a Laue photograph depends upon the arrangement of the atoms within the crystal employed. If, then, such a photograph is taken with a magnetic crystal, the character of the diffraction pattern should change when the direction of magnetization of the crystal is altered." Inasmuch as the phenomenon of heat is so closely associated with the molecules, and heat causes the disappearance of magnetism in a magnetized bar of steel, this experiment of Compton and Trousdale will be seen to be of importance in the gradual building up of our theories of magnetism. In the experiments just described, no effect of this character was found. "The obvious conclusion is that the ultimate magnetic particle does not consist of any group of atoms such as the chemical molecule." This reduces our magneton to the atom or some part of the atom.

Compton (A. H.) and Rognley[8] next made a crucial test on this point. It was known that the relative intensity of the different orders of an X-ray spectrum line depends upon the distance of the electrons from the middle planes of the atomic layers in the diffracting crystal.[8]

Now suppose that X-rays are reflected from the surface of a ferromagnetic crystal composed of atoms of the Bohr type. "When the crystal is unmagnetized the different atoms will have their electronic orbits scattered in all possible planes, so that on the average the electrons will be at an appreciable distance from the mid planes of their atomic layers. If, however, the crystal is magnetically saturated perpendicular to the reflecting face of the crystal, the electronic orbits, being perpendicular to the magnetic axes of their atoms will all be parallel to the crystal face. The electrons will, therefore, now be *in* the mid planes of the layers of atoms which are effective in producing the reflected beam." It can be shown that such a shift of the electrons must produce a very considerable increase in the intensity of the reflected beam of X-rays. On the other hand, if the crystal is magnetized

[7] Compton, K. T. and Trousdale, X., *Phys. Rev.*, Vol. 5, p. 315, 1915. Compton, A. H., and Rognley, X., *Phys. Rev.*, Vol. 9, p. 29, 1917; *Science*, Vol. 46, p. 415, 1917; *Phys. Rev.*, Vol. 16, p. 464, 1920.

[8] Compton (A. H.), *Phys. Rev.*, Vol. 9, p. 29, 1917.

parallel to the reflecting face, the turning of the orbits will carry the electrons farther, on the average, from the middle of their atomic layers, and a decrease in the intensity of reflection should result. Of course, if the electrons are arranged isotropically in the atom, or if the atom is not rotated by a magnetic field, indicating that it is the electron or the positive nucleus that is the ultimate magnetic particle, no such effect should be observed.

Compton (A. H.) and Rognley worked with two similar crystals of magnetite, and in such a way that a beam of X-rays from the same source fell on each crystal. The rays as reflected then passed through two ionization chambers, which were so arranged that if the two beams gave equal ionization in their respective chambers, no deflection of the electrometer occurred. It was a null method. When the two beams had been balanced for equality of ionization, then one crystal could be magnetized parallel and the other normal to the surface of reflection, and vice versa. The authors hunted in vain for a difference between the two beams, but could not find any. As they said (1917) "The most obvious explanation of our negative result is that it is not the atom which is the elementary magnet, but that it is either the positive nucleus, as suggested by Merritt, or the electron, as suggested by Parsons." Thus, from their negative results, it was not possible to explain them on the basis of an atom acting as a whole in the capacity of an elementary magnet.

From auxilliary evidence these two experimenters were led to favor the negative electron as the ultimate magnetic particle.

These two experiments thus narrowed down the hunt for the elementary magnet to something beyond the molecule or the atom, i.e., to something within the atom. They were, therefore, important factors in developing our theory of magnetism.

Gyromagnetic Effect. Although the electron was selected as the ultimate magnetic particle, the problem still remained whether its magnetic properties were due to its motion in an orbit about the positive nucleus—a planetesimal concept of the atom—or was its magnetic moment due to the fact that it was an electric charge spinning like a top about its own axis, or possibly a combination.

At the time of the two experiments just discussed, there was already under way an experiment which may be rated second only in importance to Oersted's discovery in the development of magnetic theories. On account of the difficulties of the experiment in its early states, the results did not yield sufficient evidence to answer the last question proposed above concerning the negative electron as the ultimate magnetic particle.

In 1908 Richardson[9] put forth a theory that if an electron is re-

[9] Richardson, *Phys. Rev.*, Vol. 26, p. 248, 1908.

volving about a central nucleus in an orbit or spinning on its own axis, it would possess a certain moment of momentum, the value of which in one case would be different from that in the other.

Hence, if the elementary magnets in a cylindrical bar of iron were turned in every possible direction and then suddenly subjected to a longitudinal magnetizing force, the bringing of all these revolving electrons with their axes of revolution, as well as the spinning electrons, into a common direction, would establish a resultant moment of momentum which would be conserved by the cylinder of iron being given a mechanical rotation in a direction whose moment of momentum would just balance that of the revolving, or spinning, electrons. Inasmuch as the moments of momentum for revolving and for rotating electrons are different, we have here a crucial experiment for determining whether the ultimate magnetic particle revolves about a central nucleus, or rotates on its own axis.

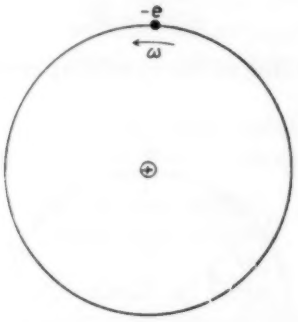

FIG. 47. An electric charge, $-e$, and mass, m, revolves about a positive nucleus. R = Radius of orbit.

In order to determine just how crucial this experiment becomes, we need to see the theory which lies back of it. It is evident at once that from the direction of the applied magnetizing force and the direction of rotation of the cylinder of iron, and knowing the masses of the proton and the electron, we shall be able to ascertain with certainty which one is participating in the effect.

Richardson tried to find this effect experimentally, but was not able to do so because he could not eliminate disturbing effects. Furthermore, his calculations were made at a time when the idea of an electron revolving about a central nucleus held the stage, so that he was concerned only with the case of a revolving electron. In the course of the development of the theory of this effect—which is now called a *gyromagnetic effect*—it was found that the ratio between the angular momentum Z developed, and the magnetic moment M was

some function of the ratio m/e, where m is the mass of the revolving charge and e its electric charge.

As the first case let us calculate the ratio Z/M for a revolving charge e and mass m. See Fig. 47.

$$\frac{2\pi}{\omega} = T,$$

the period of revolution of the charge.

Also

$$\frac{\text{charge}}{\text{time}} = \text{current}$$

or,

$$\frac{e}{T} = C = \frac{e\omega}{2\pi}$$

$$A \times C = M,$$

the magnetic moment due to the revolving charge.

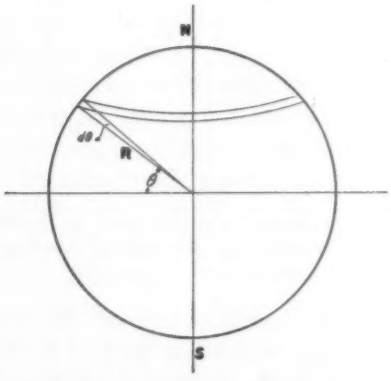

FIG. 48. A sphere with surface charge uniformly distributed rotates on its axis N-S.

$$M = \pi R^2 \left(\frac{e\omega}{2\pi}\right) = \frac{e\omega R^2}{2}.$$

On the other hand the angular momentum

$$Z = I\omega = mR^2\omega$$

$$Z/M = \frac{mR^2\omega}{e\omega R^2/2} = \frac{2m}{e}.$$

This ratio is called the gyromagnetic ratio, $(=G)$, and can be measured. We can learn from it a great deal about the ultimate magnetic particle.

Einstein and deHaas[10] were the first to announce positive results, and they found a value corresponding to that deduced above. However, Barnett,[11] who has done a prodigious amount of work on this subject and its converse, by which a ferromagnetic body becomes a magnet by the simple means of rotating it, found that

$$\frac{Z}{M} \text{ was nearly equal to } \frac{m}{e}.$$

His final results for iron gave

$$G=\frac{Z}{M}=1.031\ \frac{m}{e}.$$

This indicated that the ultimate magnetic particle was not a revolving electron, but a spinning, spherical, electron, uniformly and negatively charged on the surface, an assumption made by Lorentz and strongly supported by the classical experiments of Kaufmann and of Bucherer.

Consider the sphere m, Fig. 48, with a uniform surface density of charge ρ. Let dA be the area of a strip about the sphere, such that

$$dA=2\pi R\cos\theta(Rd\theta).$$

The charge on this strip,

$$dQ=[2\pi R^2\cos\theta d\theta]\rho.$$

When it revolves, the current C will be

$$C=\frac{\omega}{2\pi}[2\pi R^2\cos\theta d\theta]\rho.$$

The Magnetic Moment

$$dM=CA=\frac{\omega}{2\pi}[2\pi R^2\cos\theta d\theta]\rho(\pi R^2\cos^2\theta).$$

The total magnetic moment due to the entire surface charge is

$$M=\pi\rho\omega R^4\int_{-\pi/2}^{+\pi/2}\cos^3\theta d\theta$$

$$=\pi\rho\omega R^4\left[\sin\theta-\frac{\sin^3\theta}{3}\right]_{-\pi/2}^{\pi/2}$$

[10] Einstein and deHaas, *Verh. d. D. Phys. Ges.*, Vol. 17, p. 152, 1915, Vol. 18, p. 173, 1916.

[11] Barnett, Fac. Research Lecture at the Univ. of Calif., Los Angeles, delivered April 20, 1928.

$$= \tfrac{4}{3}\pi\rho\omega R^4 = (4\pi R^2\rho)\omega R^2/3$$

but

$$4\pi R^2\rho = e,$$

the elementary charge, and therefore

$$M = \tfrac{1}{3}e\omega R^2$$

and

$$Z = \tfrac{2}{5}mR^2\omega$$

$$G = \frac{Z}{M} = \frac{\tfrac{2}{5}(mR^2\omega)}{\tfrac{1}{3}(e\omega R^2)} = \frac{6}{5}\,(m/e) = 1.2\,\frac{m}{e}\,.$$

This is on the assumption that there is no deformation of the sphere due to spin. Suppose that the sphere rotates until it is nearly a disk in shape, and all the charge is on the surface, not taking into account the ends of the disk. Such a cylinder spinning would have a gyromagnetic ratio such that

$$G = \frac{Z}{M} = \frac{m}{e}\,.$$

Where Z and M for the disk are respectively

$$Z = \tfrac{1}{2}m\omega R^2 \quad \text{and} \quad M = \tfrac{1}{2}\omega eR^2.$$

The experimental value for G is so far from the value $2(m/e)$ that it practically leaves the ultimate magnetic particle to consist of a spinning charge, viz., the negative electron.

This experiment becomes in the field of magnetism the most important one in succession to Oersted's original discovery, which opened up the idea that magnetism was due to electric charges in motion.

So important is this experiment that it must now be carried out with the greatest possible precision. It is difficult because the ultimate in precision and refinement must be used in order to find out as much as possible about the character of the spinning negative electron. To this end, Scott,[12] at the General Motors Research Laboratories, is carrying out a very extensive and refined set of abservations on the gyromagnetic ratio, G, for iron, nickel, cobalt, and some alloys.

For iron he found $G = 1.0278 \pm .0014\ (m/e)$. In view of the fact that he is working with extreme precision and has at his disposal the resources of the General Motors Research Laboratories, we may look

[12] Scott, *Phys. Rev.*, Vol. 28, p. 542, 1951.

forward to some results which will throw more light on this subject. The question still remains, why G is not exactly m/e. Is it due to the fact that a part of the magnetic force we observe in iron, for instance, may be caused to a slight extent by the orbital movements of the negative electrons, or do we have deformation of the charged spherical electron to the extent that it approaches $G=m/e$? The spinning sphere, it must be remembered, gave $G=1.2$ (m/e) and the experimental value is 1.0301 m/e, while for a disk $G=m/e$.

The Magnetic Moment of the Spinning Electron. Thus, through a series of crucial experiments we have been led to the idea that the ultimate magnetic particle, the magneton, is the negative electron spinning on its own axis, and has a definite magnetic moment equal to $M=0.92\times10^{-20}$ cgs units.[13]

For a spinning electron its angular momentum,

$$p=\frac{h}{4\pi}$$

where h is a quantum constant.

$$\frac{M}{p}=\frac{e}{mc}\ (c=\text{velocity of light})$$

or

$$M=\frac{pe}{mc}=\frac{eh}{4\pi mc}$$

$$=\frac{4.8\times10^{-10}\times6.62\times10^{-27}}{4\times3.1416\times9.11\times10^{-28}\times3\times10^{10}}$$

$$=\frac{31.776\times10^{-38}}{343.265\times10^{-18}}=0.92\times10^{-20}\text{ cgs units}$$

where e is given in electrostatic units, i.e., in stat-coulombs.

As we measure more and more accurately the gyromagnetic ratio, the more we shall know about the magnetic moment of any given atom. In general, the magnetic moment of an atom consists of two parts, the orbital contribution and the electron spin contribution. There can be no question about the fact that the accurate measurement of the gyromagnetic ratio constitutes one of the major experiments in the field of magnetism today.

The magneton, no matter what its ultimate character, will have a definite magnetic moment—it is a dipole having definite polarity. While the magnetic poles may be difficult to locate and are not con-

[13] Selwood, *Magneto Chemistry*, p. 74, 1943. Interscience Publishers, Inc., New York.

sidered fundamental entities, the magnetic moment of a magneton is a very real thing.

Magnetic Shells. Because the magneton has a reality, there will come a day when we can express it *simply* in terms of the equivalent current of the electron spin and its resultant area. When we can do this, the constant of a magnet may be definitely expressed in the same terms, i.e., in terms of moving electric charges.

Ampere discovered this general relationship in studying the magnetic fields about a conductor. He set it forth in a general theorem: "*Every linear conductor carrying a current is equivalent to a magnetic shell, the bounding edges of which coincide with the conductor, and the moment of which, per unit area,* that is, the strength of the shell, *is proportional to the strength of the current.*

By a magnetic shell is meant, in modern concepts, an infinitely thin single layer of dipoles which are spinning electrons, and whose axes of spin are normal to the sheet, so that one side of the sheet is a N, and the other a S polar surface. The thickness of the sheet dt is the polar length of the spinning electron.

Ampere's proposition may be derived in the following way: In Chap. III, p. 11, it was shown that the force along the axis of a single coil at a distance S from the center of it, was

$$H_P = \frac{2\pi R^2 C}{(S^2+R^2)^{3/2}}$$

or, when S was large with respect to R,

$$H_P = \frac{2\pi R^2 C}{S^3}.$$

Suppose now that we replace this coil with a magnet at a point which was the center of the coil, the strength of whose pole is N and whose length is dS, the magnetic axis lying along the line S in Fig. 36. Let this magnet exercise the same force on a unit pole at P, Fig. 36, as did the current flowing in the coil. The moment of the magnet is NdS. Let the pole of the magnet turned toward P be such as to attract the unit pole at P. The magnitude of this force will be

$$H_P = \frac{N}{S^2}$$

while the further pole will repel the one at P with a force

$$H_P' = \frac{N}{(S+dS)^2} = \frac{N}{S^2} - \frac{2NdS}{S^3}.$$

approximately. The total force at P due to the magnet will be

$$H_{P_t} = H_P - H_P' = \frac{2NdS}{S^3}$$

while that due to the coil of equal amount was $2\pi R^2C/S^3$. Therefore

$$\frac{2\pi R^2C}{S^3} = \frac{2NdS}{S^3}$$

or

$$(\pi R^2)C = NdS.$$

Where $\pi R^2 = A$, the area of the coil, and $NdS = M$, the magnetic moment of the magnet, and C the current, or

$$M = A \times C,$$

a very important equation in our modern theories of electricity and magnetism, which was discovered by Ampere over 125 years ago.

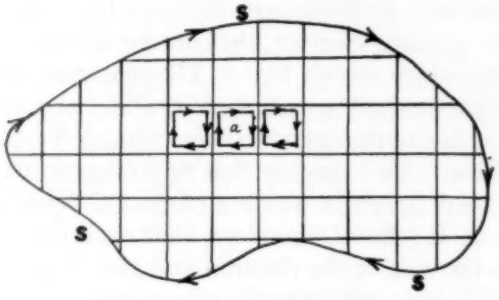

FIG. 49. A magnetic shell.

If dS is the thickness of the shell, and dA is the area of the element chosen, for instance of a magneton, then $dA \times dS$ is equal to the volume it possesses.

Let θ equal the pole strength of each unit area of the magnetic shell, and $\theta dA dS$ will be the magnetic moment of the element contained in the volume $dA dS$. The product θdS is called the strength of the shell, which we may designate by the symbol ϕ. It is the magnetic moment per unit area of the shell.

$$\phi = \theta dS.$$

If, in a unit area of the shell, there are σ unit magnetic poles, then

$$\phi = \theta dS = \sigma dS$$

or

$$M' = \phi dA$$

the magnetic moment per element of the shell. On making the area that of the whole shell, then

$$M = \phi A = CA$$

and the strength of the magnetic shell is numerically equal to the current flowing around the periphery of the shell, which is exactly what was said in the original proposition of Ampere (p. 22).

In order to help visualize some of the foregoing discussion, consider the circuit *SSS* in Fig. 49 as being traversed by a current *C*. We can imagine this circuit as being replaced by an infinitely large number of small elements like *a*, each of which is traversed by the same current *C*. For each boundary line between the two elements, it will be seen that there are two currents flowing along it, but in opposite direction, both of current strength *C*. These all neutralize each other, and only in the elements of which the contour of *S* forms one side, is the current not neutralized. In the end, only the current in the contour line of *S* is effective and unneutralized.

Next consider each little element replaced by a tiny magnetic dipole such as the spinning electron, the contour of which coincides with that of the elementary circuit like *a*. The magnetic moment of this elementary magnetic shell is such that its effect at a distant point is the same as that due to the current element which it replaces.

To recapitulate: This is saying that any magnet may be replaced by a certain closed circuit in which a current *C* flows, and whose effective area is *A*. It follows, therefore, that once we know the effective current in the spin of the electron and the effective area of that current we can express all magnetic phenomena in terms of electric charges in motion. This is the goal toward which Ampere worked.

Experimental evidence has shown in this chapter that magnetism is produced by electricity in motion, and that the fundamental unit of magnetism is the spinning electron, which produces a magnetic field just as a circular loop of wire does in which an electric current flows. Furthermore, these tiny whirls of electricity, when subjected to a sufficiently strong magnetizing force, turn so that their magnetic fields are parallel to the applied field. How are the properties of matter affected in which these whirls are located, when these orientations of the spinning electrons occur? The succeeding chapter will try to answer one important phase of this question. Various effects are observed when the orientation of spinning electrons are brought about by an applied magnetizing force, but change in what we call the magnetic induction of a substance is most important from a practical as well as from a theoretical point of view.

GEOMETRY OF PAPER FOLDING

I. PROPERTIES OF A DRINKING CUP

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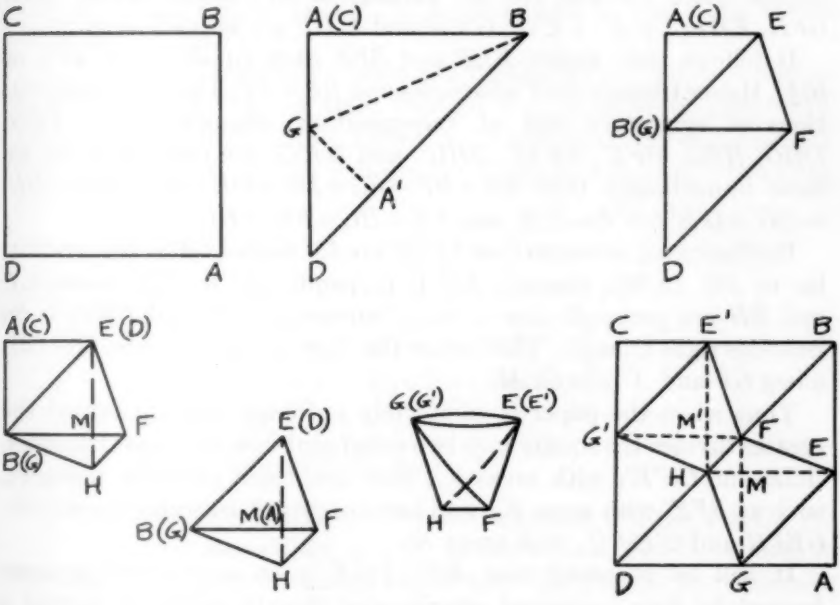
On page 24 of his *Fun With Paper*, F. A. Stokes Co., (1940), Joseph Leeming tells how to make a drinking cup by folding a paper square. It is our purpose to modify his instructions, and to point out some of the interesting relationships inherent to the configuration formed by the creases which result from the folding.

Forming the Cup. A square $ABCD$ may be folded into a cup by proceeding as follows:

1) Bring A over into coincidence with C and flatten out the paper so as to crease along the diagonal BD, forming two superposed congruent isosceles right triangles. The notation, $A(C)$, in the figure indicates that A lies above C.



2) Bring B over to G, on AD, and flatten out the paper, creasing along EF so that BE is parallel to DF. An alternative direction would require that BF be perpendicular to AD. Either of these operations is readily accomplished with a fair degree of accuracy. However, if more precision is required, G can be located definitely by

* Mr. Trigg will present a series of short papers for our readers. These were requested by both the Editor and the Mathematics Editor, Professor Read.—G.W.W.

bringing AB over to coincide with BD. When the paper is flattened, the crease BG bisects angle ABD.

3) Bring D around under the paper to coincide with E, and flatten out the paper to form the crease GH.

4) Fold the triangular flap AEG up along EG and tuck it into the open edge BE.

5) Fold the triangular flap $CE'G'$ down along $E'G'$ and tuck it into the open edge DG'. (The prime notation indicates that after operation 1), G' lies directly below G.)

The cup so formed has an open edge between GE and $G'E'$. It is easily held in one hand with the bottom HF supported by the little finger, and opens up when light squeezing pressure is exerted on GH and EF. A little contemplation will convince one that the cup is quite stable, is sealed up to the level of the rim, and provides a very satisfactory drinking cup.

The Pattern of the Creases. Since triangles ABD and CBD are isosceles right triangles, angles ABD, ADB, CBD and CDB are 45° angles. Since BE and DG are parallel to HF, angles AGE, AEG, GFH, EHF, $CG'E'$, $CE'G'$, $G'FH$ and $E'HF$ are also 45° angles.

It follows that angles BEF and BFE each equal $\frac{1}{2}(180°\text{-}45°)$ or $67\frac{1}{2}°$. Hence triangle BEF is isosceles and $BE=BF$. Then by considerations of symmetry and of superposition, triangles BFE, FEG, DHG, HEG, BFE', $FE'G'$, DHG', and $HE'G'$ are congruent. So we have immediately that $BE=BF=EG=DG=DH=GF=EH=BE'=DG'=G'E'=G'F=E'H$, and $FE=HG=FE'=HG'$.

Furthermore, since AGE and EGF are 45° angles, GF is perpendicular to AG. In like manner, EH is perpendicular to AE. Hence GF and EH are perpendicular at their intersection M, and FMH is an isosceles right triangle. Then when the flaps are folded over, AG falls along GF and A falls on M.

Thus when the paper is completely unfolded, the pattern of the creases divides the square into two congruent isosceles right triangles, AGE and $CG'E'$, with areas K_1; four congruent isosceles triangles, such as BFE, with areas K_2; and two congruent isosceles trapezoids, $GEFH$ and $G'E'FH$, with areas K_3.

It will be observed that $BE'G'DGE$ is an equilateral hexagon formed by four congruent overlapping rhombi arranged around a square. This becomes clear when the traces of the folded portions of DB are drawn in, that is GF, EH, $G'F$, and $E'H$. It is evident also that the square joins two larger squares to form a symmetrical pattern.

Lengths of the Creases. Consider $ABCD$ to be a unit square, and denote AE by x. Then $EB=EG=x\sqrt{2}$, so $x+x\sqrt{2}=1$, and $GM=ME=AE=x=\sqrt{2}-1$, while $EG=BF=DH=GF=2-\sqrt{2}$. It

follows that $FM=GF-GM=(2-\sqrt{2})-(\sqrt{2}-1)$ or $3-2\sqrt{2}$, so $HF=FM\sqrt{2}=3\sqrt{2}-4$. Otherwise, $HF=DB-BF-DH=\sqrt{2}-2(2-\sqrt{2})$ or $3\sqrt{2}-4$. Furthermore,

$$FE=\sqrt{(FM)^2+(ME)^2}=\sqrt{(3-2\sqrt{2})^2+(\sqrt{2}-1)^2}=\sqrt{20-14\sqrt{2}}.$$

When AB was folded over onto BD so that A fell on A', $A'G$ was perpendicular to BD, so $AG=GA'=A'D=BD-A'B=\sqrt{2}-1$.

Relationships of the Areas. ME is an altitude of both triangles EMG and EFG and consequently $K_1:K_2=GM:GF=1:\sqrt{2}$. Trapezoid $GEFH$ = triangle HEG + triangle FEH = triangle BFE + triangle FEH = triangle BEH. Therefore

$$K_2:K_3=BF:BH=(2-\sqrt{2}):(2-\sqrt{2}+3\sqrt{2}-4)=1:\sqrt{2}.$$

Thus $K_1:K_2:K_3=1:\sqrt{2}:2$, which stated otherwise is $K_2=\sqrt{K_1K_3}$.

Denote the areas of the squares on AB, AE, MF by P_1, P_2, P_3, respectively. Now $P_1=(AB)^2=1$, $P_2=(AE)^2=(\sqrt{2}-1)^2=3-2\sqrt{2}$, and $P_3=(MF)^2=(3-2\sqrt{2})^2$. It follows that $P_2=\sqrt{P_1P_3}$, indeed $AE=\sqrt{(AB)(MF)}$.

Trigonometric Functions. Since BG bisects the 45° angle ABD, we have immediately in the right triangle BAG that angle $ABG=22\frac{1}{2}°$ and angle $AGB=67\frac{1}{2}°$. Now $AB=1$, $AG=\sqrt{2}-1$, so $BG=\sqrt{4-2\sqrt{2}}$. Then without use of the half-angle or sum-of-two-angles formulae, we have

$$\sin 22\tfrac{1}{2}^c=\cos 67\tfrac{1}{2}°=(\sqrt{2}-1)/\sqrt{4-2\sqrt{2}}=\tfrac{1}{2}\sqrt{2-\sqrt{2}}$$

$$\cos 22\tfrac{1}{2}°=\sin 67\tfrac{1}{2}°=1/\sqrt{4-2\sqrt{2}}=\tfrac{1}{2}\sqrt{2+\sqrt{2}}$$

$$\tan 22\tfrac{1}{2}°=\cot 67\tfrac{1}{2}°=(\sqrt{2}-1)/1=\sqrt{2}-1$$

$$\cot 22\tfrac{1}{2}°=\tan 67\tfrac{1}{2}°=1/(\sqrt{2}-1)=\sqrt{2}+1$$

$$\sec 22\tfrac{1}{2}°=\csc 67\tfrac{1}{2}°=\sqrt{4-2\sqrt{2}}$$

$$\csc 22\tfrac{1}{2}°=\sec 67\tfrac{1}{2}°=\sqrt{4+2\sqrt{2}}.$$

The same values could have been secured from triangle FME.

The Incommensurability of $\sqrt{2}$. If the square $ABCD$ is made of foolscap or ruled penmanship paper with the rulings perpendicular to AD, then G may be easily located by lining up BF parallel to the rulings. If the rulings are parallel to AD, then BF is lined up perpendicular to the rulings.

Now if the distance between the rulings be unity, then the distance between the intersections of the rulings with BD is $\sqrt{2}$. If, after folding, one of these intersections along BF happens to fall on a ruling parallel to AD, certainly no others will, and we have an intuitive proof of the incommensurability of $\sqrt{2}$.

SCIENTIFIC METHOD AND SOCIAL SCIENCE

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Taking a broad overview of the natural sciences on the one hand and the social sciences on the other, one is struck with an amazing contrast. Man's understanding and control of physical nature is enormously more advanced than his understanding and control of social relations. In the past thirty years, physics and chemistry have made startling progress—in electronics, atomic energy, plastics, synthetics. At the same time our knowledge and control of social relations seems to be virtually at a standstill. In fact where there has been progress in the social field, as in improvements in the plane of living, it is held by many to be due almost entirely to science and technology rather than to any deeper knowledge with respect to economic and political relations as such.

During World War I, we had two slogans: It was to be a war to end war and a war to make the world safe for democracy. Where are these slogans today? How far do we seem to have advanced over the outlook we had a generation ago? Our social outlook is obviously more pessimistic now than it was then.

Why does this amazing contrast exist? During the early 30's, Jerome Michael and Mortimer Adler wrote a treatise on criminology following a three-year study by a group of experts. Their main conclusion, because of the backward condition in which they found criminology, was that criminologists and other social scientists were simply incapable of doing scientific work! Was this accusation justified? Some years before that, Pitirim Sorokin, Professor of Sociology at Harvard, wrote a book on contemporary sociological theories in which, on the contrary, he gave a number of examples of mathematicians, physicists, chemists, biologists, eminent in their own special fields, going over into the social sciences and making a mess of things there. What these natural scientists did primarily was to bring analogies into the social studies (mechanistic and organismic) which have, if anything, held back the social sciences rather than giving them a thrust in a forward direction. The answer to the puzzle is, in part at least, that social relations are much more complex and fluid than are physical and biological phenomena.

But before going further, let us examine the nature of the scientific method as used in the physical and biological sciences. Broadly speaking, scientific method may be said to be the unbiased objective search for truth. It is still an open question whether ultimate truth comes from within man or from without, that is from physical nature. Natural scientists have long since decided that their task is to deter-

mine the facts about nature, but of course the mind of man is not left out in the search for those facts.

It is today recognized that the scientific method is dual in character. It is observation or facts on the one hand, theory or hypothesis on the other. Which comes first and which is more important? This may seem like the chicken versus the egg argument. In one sense it is. But, depending upon the particular stage of an investigation or the kind of investigation, we get the answer as to which approach is the more important at a given time—the search for facts or an adequate hypothesis regarding what facts are already known. Still, it may be argued that the gathering of facts must always be more important, more objective, than theories about the facts. Let us see.

For a long time what may be called a shuffled-pack-of-facts approach was in vogue. It was thought that all the facts were "out there" somewhere. The main task of the natural scientist seemed to be to go out, gather an armful of facts, come back into his study and arrange them neatly into preordained or preconceived suits. Then in the seventeenth century, a series of penetrating investigations were made by physicists into the phenomena of sound and color, after which it came to be recognized that sound as we hear it or color as we perceive it, is not the same as the physical activities which give rise to these sensations. Some people cannot perceive differences in color. We say they are color-blind, which serves to illustrate the fact that what the mind perceives and what is "out there" in nature may be quite different. These discoveries, rather disconcerting for the physicist and the philosopher, started a long train of controversy which continued well into the eighteenth century. After all, seeing, smelling, hearing, and touching mean observing. To have observation made partially subjective instead of wholly objective brought about a change in the concept of absolute objectivity. A distinction finally came to be drawn between secondary qualities, such as color and sound and smell, and primary qualities in nature.

The primary qualities are extension, form, and motion. Physicists continued to be interested in primary qualities particularly, and these were still regarded as being "out there" in the objective universe. It was in connection with the controversies of the period that Samuel Johnson is said to have put forth his famous "when" definition with respect to matter. He was asked, "What is matter?" and he replied, "Why matter is *when* you kick a stone." And there the question of the objective versus the subjective in large part remained until the end of the last century.

It was during the 1890's that another important change in outlook took place. This resulted from the discovery of the electron, X-rays, and radioactivity. One can hardly exaggerate the influence of these

advances upon scientific methodology, especially in physics and astronomy. The structure of the atom as it has since been conceived is so fantastic as to throw completely out of gear earlier conceptions of extension, form, and motion. A table seems to be stable—to have extension and solidity—and yet modern physics tells us that there is much more open space than solid matter here, that the table is composed of an endless number of solar systems, a boundless amount of energy with electrons moving with incredible speeds around their central protons and neutrons, and that the relation between the sizes of a central nucleus and its electrons and the spaces intervening is similar to the relation between the size of the sun and the planets and the spaces between them in the skies. And yet it is such "out-of-this-world" theories of electronic behavior that have led to the radio, the talking picture, television, and to other Alladin-like electronic devices developed during World War II, not to mention the even more fantastic developments of chain-reacting atomic fission. The effect upon scientific method has been to narrow the distinctions still more between the objective and the subjective. Even extension and form are not what they seem, and the old shuffled-pack-of-facts theory has become completely passé in the physical sciences.

Modern scientific methodology has taken on a question-and-answer character. For example, three men enter a room in which there is a table and a typewriter. One man becomes interested in the pictures and curtains in the room. The second sees that the table is an antique and he examines it rather closely. The third is interested in typewriters and notices that this one is a Remington Noiseless. On leaving the room they engage in a discussion. The first has seen neither the table nor the typewriter; the second noticed the table but not the typewriter; the third has observed the typewriter and realizes it must have been resting on something but has not noticed the table. The "facts" were there for all three to see, and by swapping impressions they can now go back and get a better idea of table plus typewriter than any one of the three had before.

If you ask nature a poor question, you get a poor answer. If you reformulate your question to better effect, you get a better answer, and so on indefinitely. Similarly, in economics and other social fields. The better the questions you are able to formulate, the more adequate will be the answers you obtain. Scientific method still remains a dual process—theory or question on the one hand, answer or fact on the other. But it is no longer naively assumed that all facts are completely "out there" and all theories completely "inside" of us.

Scientific questions or theories must, of course, be directed to a better understanding of the facts, not to a rationalization of preconceived notions or prejudices. Not as with the pre-Darwinian natural-

ist who was content to pigeonhole his bugs according to the old Linnaean classification. A fellow naturalist who had become suspicious of such classification, asked his friend one day, "What do you do with a bug when you find it doesn't fit neatly into one of these pigeonholes?" The pre-Darwinian answered, "Well, I crush the pesky critter under my foot!" It was just such "pesky critters," however, which later led to the evolutionary hypothesis. Darwin put to nature a startlingly new question and he spent twenty years of fact-gathering to secure a satisfactory answer.

The Brookings Institution's four volumes on "Income and Economic Progress," published in the mid-thirties, illustrate the modern scientific method applied to economic problems. Throughout these volumes one finds a broad theory or question, unorthodox in character. Available facts are submitted to test out the question and other theories are examined and found wanting. The results are still being debated and further evidence is being sought.

You cannot have adequate fact-gathering without a satisfactory theory. Nor is it a satisfactory procedure to brush certain facts aside as "pesky critters" that get in the way of "received" or preconceived notions.

We turn now to the question, "In what respects are the social sciences more complex and fluid than the natural sciences and thus more difficult to formulate?" We may start by asking a still broader question, "What is a society, and on what does our understanding of it depend?" Broadly speaking, one may say a society is an organized body of interacting members—individuals, families, races, classes, nations—bound together by common desires and customs around which institutions have developed. Our understanding of such an organized "body" depends in part on our knowledge of its environment. At this point we get assistance from the natural sciences, from physics, chemistry, astronomy, geology, biology. Our understanding will also depend, and much more significantly, on the human desires and customs on which that society is based. Here we look to the sciences of psychology and anthropology, but these are not as far advanced as are the natural sciences and much still needs to be done before they can be regarded as reliable guides to constructive social change. We are still in process of fashioning various kinds of societies. Nor are we completely sure as to what types will serve human needs and desires best.

We find here the first important distinction between the social sciences and the natural sciences. Besides applying the scientific method to what social facts happen to exist, social scientist should be in a position to make recommendation for improvement. Secondly, there are both "artificial" and natural elements in a social structure.

Here the social scientist faces another problem. He must determine what elements in a given society are wholly natural and unchangeable, what elements in this society are natural but divertable to man's uses and purposes, and what elements are wholly artificial and thus completely changeable, if that seems desirable. Thirdly, moral issues are so tied up with social facts that social scientists find it difficult to withdraw themselves for an unbiased study. Otherwise it is all too easy to rationalize existing preconceptions and prejudices. Professor Ralph Barton Perry, the philosopher, writing some years ago in the *Quarterly Journal of Economics* on the subject "Economic Value and Moral Value," submitted an interesting list of preconceptions that he found implicit in economic writings. One preconception was that each factor in production should get the equivalent of what it contributes; another, that distribution should be in accordance with absolute intensity of need; a third, that strictly business valuations should be replaced by human valuations; still another, that distribution should conform to existing class distinctions, thus perpetuating a cultivated leisure class and a proletariat to do the chores; a fifth, that the commercially efficient should be allowed full freedom in the intersets of national supremacy or the survival of the "fittest." It is difficult to get away from such implied norms in the social sciences. No such problem exists with respect to physics and chemistry as pure sciences. In addition, as just indicated, after completing an unbiased study in the social field, one is still faced with the problem of making recommendation for betterment, as with bills now being considered by Congress. No one would think of asking an astronomer to recommend new movements for the planets. In economics, political science, sociology, and the rest of the social sciences, it is not enough to gather facts and to have realistic theories regarding facts. There is the further necessary objective of policy making, of improving upon the social structure that happens to exist at a given time.

And here the student of society is faced with the most perplexing and unique problem of all, as compared with the physical or biological scientist. Where is he to turn for his social objectives or norms, those judgments and values—the tertiary qualities—which are so much more difficult to formulate than the secondary and primary qualities of the natural sciences? These are indeed hard questions to answer. They remain an ever-present challenge to the student of society.

Garden sprinkling control can be set to turn on a sprinkler system at any time of day or night and to shut it off later. A special device prevents the sprinkler from going into operation on rainy days or when the earth is properly moist.

TEACHING A CERAMICS UNIT IN HIGH SCHOOL CHEMISTRY

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Not only is the chemistry of clay, pottery and glass very interesting but the necessary materials are readily available and many of the products of the ceramics industry are everyday well known household objects. In introducing such a unit in high school chemistry certain facts concerning the importance of the industry can be discussed:

(1) A dollar's worth of ceramic products are necessary for the production of every ton of steel.
(2) The annual value of the products of the ceramics industry is one and three-quarters billion dollars.
(3) It is one of the oldest manufacturing industries in the world and still growing.
(4) The use of scientific personnel in the ceramics industry is increasing and schools are offering degrees in ceramics engineering.

Several movies are available on the history of ceramics and on ceramics as an art as well as a number on the glass industry. Movies on cement can also be obtained. The chemistry of cement and concrete are closely allied but somewhat different from the chemistry of clay and can be included in the unit.

A four week unit was recently taught in the Teachers College High School, Cedar Falls, Iowa. A brief outline of the program follows:

A. Introduction and motivation
   Film: "The Art of Kerimos"
   Discussion and listing of ceramic products
   Bulletin board display of ceramic products
B. Laboratory
   1. Both native clay and china clay (kaolin)
      a. Grinding, sifting and mixing with water
      b. Molding: every student made at least one simple form.
      c. Drying: per cent of shrinkage calculated and the ability to regenerate clay after drying noticed indicating no chemical change.
      d. Firing of the clay (the kiln in the art department was used): additional shrinkage noticed and the inability to regenerate clay indicating that a chemical change has taken place.
C. Demonstration of the art of pottery making
   (Done by the ceramics instructor in conjunction with the use of the art department kiln)
D. Library research
E. Discussion
F. Laboratory
   1. Making of glass using a low fire glaze formula
   2. Borax bead tests
   3. Etching of glass
   4. Permanent markings on glass
   5. Water glass experiments
G. Library research
H. Movie: "Crystal Clear"

I. Discussion
J. Laboratory
Formulating and applying glaze to the student pottery and returning to kiln for firing
K. Field trip to Sheffield Brick and Tile Co.
L. Test

In the event a kiln is not available outdoor firing can be done. Directions for outdoor firing can be found or the description of primative methods may be followed.

It is noticeable that experience always precedes theory. This is the realistic approach to scientific laboratory work. There is no discovery possible if the outcome of an "experiment" is a foregone conclusion. Only when, as a result of tying previous experience to theory, certain additional predictions can be made should experiments testing these theories be made.

The above is not necessarily the best program for a ceramics unit but is a first attempt and proved to be very satisfactory. Noticeable results of student activity in this unit are:

1. A thorough understanding of the simpler aspects of the chemistry of ceramics,
2. An appreciation of the size and scope of the ceramics industry,
3. A greater knowledge of the simple everyday ceramics products,
4. A more critical attitude as consumers of ceramic products,
5. A greater awareness of the interrelationship of history, art and chemistry,
6. An introduction to the vocational possibilities in the ceramics industry,
7. An introduction to the hobby possibilities in the art of ceramics,
8. An appreciation of the place of chemistry in industry and the job of scientific personnel,
9. The feeling of successful accomplishment,
10. Development of laboratory techniques, reading, writing, listening and other skills.

ADELPHI ORACLE MOVES TO HARVARD NEXT FALL

Gerassimos ("call me Mike") Karabatsos (129 Roxbury Road, Garden City) has been awarded a teaching fellowship for doctoral studies at Harvard University. Next September marks his starting date with the "Crimson."

Mike is now a senior at Adelphi and a student assistant in the Research Service on campus. In addition to being editor of the college *Science Journal*, he is New York Gamma Chapter President of Alpha Epsilon Delta, a national honorary fraternity; recording secretary for Gamma Sigma Epsilon, a chemical honorary frat; and is listed in *Who's Who in American Colleges and Universities.*

Last month (March) Mike received the student medal award for excellence in scholarship, character and leadership from the American Institute of Chemists. Carl M. Herstein, president of the New York Section of the society, presented the medal at a special dinner held in the private dining rooms of Union Carbide and Carbon Chemical Co., N. Y. Mike was also made an associate member of the American Chemical Society that evening.

Rainy day outfit for men features a brown plastic ensemble of raincoat with cape back, a hat protector and leggings. The leggings, designed to keep spring showers from soaking trousers, slip on easily and are held up at the top by a loop over the belt.

THE PLAY OF THE IMAGINATION IN MATHEMATICS*

WILLIAM DAVID REEVE

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It is doubtful whether any of us ever liked a subject because a teacher somewhere or other told us we were going to use it. In fact, the "bread and butter" value of mathematics is very small. Moreover, we might as reasonably expect to interest a child in English by making him study Webster's Dictionary as to expect him to like mathematics because he is studying a mathematics textbook, written by some "humdrum" author. As Professor David Eugene Smith expressed it, "Geometry must be taught as a spiritual kaleidoscope. Turn it around and you see all sorts of beautiful things." Or, in the light of modern ideas, geometry should be taught as a motion picture. If we merely look in a kaleidoscope, the picture has only a moderate amount of interest; it is only as we give it motion that its beauty appears.

One of the methods of improving upon mere textbook teaching is to call attention to the wide use of mathematics, a fact which is apparent to everyone. If however, we seek to reveal all these uses to a body of students, we fail through the mere superabundance of material and through the lack of technical knowledge necessary to make use of what is easily available.

If we try to make the material seem less technical by means of class play, as in playing store or bank or machine shop, we run into one of the danger zones of teaching—the wasting of valuable time and the loss of that rigorous thinking which forms a vital part of the subject taught. A reasonable amount of this work is justifiable, or even highly commendable, particularly in the elementary school, but in the junior high school it is especially essential to avoid any extreme in this field. "All work and no play" is a dangerous extreme, but "All play and no work" is more so. The ideal mean which we should all seek to approach may be indicated by the words "All work should be play"—that is, it should be made just as interesting as possible.

It is well known that mathematics has many applications, but we cannot bring them all into the classroom. This was clearly shown by the futile attempts a few years ago to make geometry more interesting by an overemphasis upon its applications to church windows, tile floors and the like. It is a different thing, however, to make much of

* A paper read before the Reader's Club of the Riverside Church in New York City on Tuesday, December 15, 1953.

the beauty and grandeur of the world about us, that is, in the "clothing of mathematics."

To attain this mean we need to make the kaleidoscope move, to show to the pupils the beauty, the grandeur, and the general significance of each mathematical topic—to show the soul as well as the body, and the body as well as the mere clothing. To do this we must give to the children a freedom of play of the imagination that was generally lacking in the older type of teaching.

We cannot make it purely "child's play" because to do so would kill the science in it. That is why playing store, bank, and the like, if carried to the extreme may be fatal. In other words, the work may turn out to be all project and no arithmetic. What we need is the right kind of appreciations, attitudes, and ideals with respect to mathematics, and these can be obtained only by artistic teaching.

The trouble with our teaching in the past was that we treated mathematics as pure science and therefore made the "child's play" impossible. As Coleridge put it, "Whilst Reason is luxuriating in its proper paradise, Imagination is wearily traveling on a dreary desert."

What Coleridge said about mathematics teaching represents the older type; the modern type allows imagination an equal place with reason in the oasis. It is only in this way that we can remove the charge that mathematics is mere drudgery to many children.

There is a suggestive story told of a small boy in a drawing class. Asked to draw a picture of a man, he represented him with a very large head and with a single leg. When the teacher remonstrated with him and pointed out the defects, the reply was confidently made, "Well, that's the kind of man he is." In a large measure the child was right; he had given free rein to his imagination and to his own originality of expression.

It is evident, however, that the textbook cannot go very far in suggesting the proper play of imagination. To do so would be to tell too much, and thereby to eliminate the very thing that the teacher may properly encourage.

One way of solving the problem of arousing the student's interest in mathematics is to consider "other worlds than ours," and to handle the subject from a more human standpoint. However, the feasibility of publishing a course in mathematics that appeals to the imagination is questionable. The work of humanizing the teaching must be done largely by the teacher.

What We Study in Geometry

The geometry which we study in our schools is usually divided into two parts, plane geometry (two dimensions) and solid geometry (three dimensions), although of course we often speak of one-dimen-

sional geometry. It is unfortunate that we live in a world of three dimensions and teach, for the most part, only the geometry of Flatland. Thus, in studying the various types of geometry, the question of extending our ideas to a geometry of four dimensions naturally arises.

We are all familiar with the fact that any point in space may be determined by starting from a known point as a reference, and then measuring in three given perpendicular directions. The question, then, that naturally arises here is, may a point be determined by measuring in four given perpendicular directions from a known point of reference?

In order to understand Euclidean geometry, we draw figures and construct models, but the geometry of four dimensions cannot be so easily explained, since it deals with things which none of us has experienced. However, by analogy with our geometry of other dimensions we can conceive of situations which will help us to understand what a world of four dimensions would mean. It is not held that such an explanation is mathematical, nor that it is given for the purpose of trying to convince anyone that such a geometry exists, but merely to make clear from the properties and relations that exist in four-dimensional space, what we have in mind.

There is no point in saying that because one cannot comprehend the fourth dimension it does not exist. This proves nothing one way or the other. We pretend to know space, yet if it has no limits, we cannot comprehend it because we cannot form a concept of unlimited space; and if it has limits, we are not able to comprehend them. The mere matter of being unable to relate a situation to our own experience is no evidence that the situation is impossible. One cannot, for example, jump out of a window two-thirds of a time, and yet the fraction 2/3 has meaning. If one holds six pencils in his hand and is told to give away nine of them, this would seem as foolish to some people as the thought of a fourth dimension. Nevertheless, any child who has studied algebra has no hesitancy in taking 9 from 6, obtaining -3 for the result of the subtraction. At a time in the not far distant past, the airplane, the radio, and television, would have seemed miraculous. The whole development of non-Euclidean geometry is the result of the refusal of certain mathematicians to be satisfied with Euclid's so-called parallel postulates as the only possible basis for geometric thinking. For that matter we do not know what electricity is, although we measure it and talk about it glibly, and the same is true in regard to intelligence. Similarly, we are ignorant as to what life really is. Mathematicians formerly called $\sqrt{-1}$ an imaginary number because they could not imagine it, and yet we know today that $\sqrt{-1}$ is as real as any other number.

Professor David Eugene Smith once said:

"I know of nothing which acts as such a powerful antidote to that which I venture to call 'opinionatedness,' as a study of mathematics. To know that the light from solar systems far larger than our own has been thousands of years in reaching us gives us an idea of our infinitesimal nature, in comparison with space about us, that can come only with a study of the science that it is ours to teach. A bacillus in our veins, so small as to be invisible through a powerful microscope, is a giant compared with ourselves in our relation to this space in which we live. Our doubts, our beliefs, our hopes, our fears are all so trivial, so infinitesimal, so like a lost electron in our solar system as compared with our relative importance in the universe as revealed to us by the calculations which mathematics brings to bear upon the great problem! Cowper wrote well when he put in verse the words,

'God never meant that men should scale the heavens
By strides of human wisdom,'

and even the mathematics of youth confirm the thought."[1]

If we take a given point and imagine it to move, we say that it *generates* a line. If the line moves in any direction except along itself, it is said to generate a surface. And if the surface is moved in any direction except along itself it is said to generate a solid. Now someone may ask, "Can a solid be moved in a direction not contained in its three dimensions so as to generate a new figure whose relation to that solid is analogous to that of the solid and the surface?" If so, the new figure is the fourth dimensional figure we are trying to picture.

Pointland

In trying to understand what the fourth dimension means, let us begin by imagining what a static condition we should be in if we lived in Pointland—a land consisting of only a single point. Here a Pointlander would not be able to move (except perhaps in a whirl like the center of a revolving disc). What a cramped world this would be! Since none of us could be smaller than a point, we probably will think of only one of us in such a land. Each of us would then fill the whole of his universe. We think that we have but little room in a crowded city, although we have three dimensions (length, breadth, and height) in which to move; but in Pointland we could not move at all, although we should not realize it. Each would think of himself as the whole universe. The more narrow-minded a man gets the bigger he thinks he is. Mr. Pointlander, therefore, could not move, could not play any games, could not read or eat or drink, could not do anything that we do; for doing involves motion and he could not move at all.

Lineland

Let us imagine that in Pointland we have human beings endowed

[1] Smith, David Eugene. "Religio Mathematica," *Teachers College Record*, November, 1921, page 8.

with intellect and full reasoning powers, but who can see nothing outside of Pointland where they live. Let us further imagine that as in our world of three dimensions, a Pointlander becomes adventurous and ambitious as we often do ourselves and in some mysterious way gains control of Lineland (a world of one dimension, i.e., the path of a moving point). Mr. Pointlander could go in two opposite directions only. He would now seem very free as compared with his former self, because he could go forward and backward at will. However, he would be quite limited in his movements. If there were three points, 1, 2, and 3 in order, point 1 could never be next to point 3 because point 2 would always be between them. Communication between nonadjacent points would have to be carried on by some sort of telegraphy or radio. At best, however, conversation in such a land would be "small talk," although it might be very important in a real sense. Our imaginary creature would then be like "a fly in a lemonade straw," or "a man crawling in a sewer." Someone has referred to the germs (really cylinders) that infest our bodies as illustrating the nature of Lineland because they have length but scarcely width or height. Blood corpuscles (Linelanders) are said to have a battle royal with the invaders (the germs) and if microbes are so constituted as to produce disease, we live or die as the corpuscles succeed in the fight.

Flatland

Let us imagine further that, having become a Linelander, our former Pointlander becomes intoxicated with his freedom and power and in some way or other becomes a Flatlander. He would then be somewhat like a bee between two planes of glass, free to move but only in two directions. The inhabitants of Flatland would be as safe from observation by each other behind a line as we are behind a wall. We could see them, but they could not see us. The amoeba has been mentioned as a good illustration of a Flatlander. It has seemingly length and width and no doubt thickness as well, although it is probably not conscious of the latter. What a time a Pointlander set loose in Flatland would have! Moreover, there is no place in mathematics where the imagination can run riot as in Flatland. Speculation as to many of the conditions therein gives rise to great interest on the part of all students. Dr. Abbott's well-known book, *Flatland*,[2] is an excellent bit of source material for such a book. Some may say it is all nonsense, but any teacher who has tried to get children interested in using their imaginations knows that there is a great deal of good sense in it. The students enjoy speculating about what kinds of classrooms they have in Flatland, how the students stand in class, how they

[2] Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. Price $1, paper bound.

write on the blackboard, which is a straight line, and so on. Obviously a Flatlander would draw a plane on his blackboard in a way similar to the way we draw a picture of a box on ours.

We may conveniently think of Flatlanders as our shadows. They would have no use for such words as "up" or "down," "high" or "low." Their houses are simply a series of rectangles, or, as Dr. Abbott puts it, pentagons. With the doors and windows shut, they could not get out. In a mathematics classroom no one could go to the front of the room without climbing over everybody in front and the teacher would always have to be there first. A savings bank in Flatland would be a circle, but it would have to be a very large one, for there could be no piling up of the money in the banks of this realm as there is in ours. People in Flatland would be utterly unable to tie the two ends of an ordinary piece of twine into a knot. The students in the mathematics classes in Flatland could not make use of the method of proof by superposition in the way we use it (unless, perchance, the figures to be superimposed were properly arranged in the same order) because it would be impossible for them to pick up any figure.

If one of these shadow creatures were suddenly transported into our world and should watch one of us push an ice-cream cone down into a pail of water (apex first), he would see first a point and then a series of circles getting larger and larger and finally disappearing altogether. He would seem to us to be greatly handicapped because if he were put inside a rectangle he could not get out and yet he could enter or leave one-dimensional space at will.

Three Dimensions

Now let us assume for illustrative purposes that one of our "shadow" people beomes endowed with a knowledge of three dimensions. He could at once do wondrous things in the eyes of his fellows. He could vanish and return at will. He could not be held in the strongest prison of the land. He could see into his neighbor's houses and even into their very insides. And if, before reappearing, he should turn over he would be a sort of reflection of his old self in the eyes of his friends, that is, he would be left-handed if he had been right-right-handed before and his heart would be on his right side instead of his left (assuming that it was on the left at first). You might think he would lie on his face. This would be as comfortable as before.

Four Dimensions

Now this same person, having become endowed with a knowledge of three dimensions, might become so intoxicated with his power that he would discover a way to master the fourth dimension. Let us

assume that this is so. He could then perform the same antics before us that we have supposed him able to perform before our two-dimensional shadows. None of our prisons could hold him, he could take his departure from one of our homes without going out at a door or a window and we would not be able to tell how or when he left. He could rob a bank safe without blowing it or without even knowing the combination. He would be able to tie a knot and then pull it out with the ends of the string fastened. He could turn a football wrong side out without cutting it. Turning a right-hand glove inside out to make it fit the left hand would have the same effect as simply turning it over in the fourth dimension, just as the flat drawing of such a glove can be turned over in three dimensions. In the latter, the fingers and thumb do not come out through the wrist but every part by itself is turned over and the glove remains a right glove. A fourth-dimensional physician could perform operations upon the innermost parts of the body without even piercing the skin. Thus, operations for appendicitis would become more fashionable than ever because taking out an appendix would be much like cutting off a corn on one's toe. With such power one could see how much money his neighbor had in his pocket. It has even been suggested by someone that perhaps many articles which are lost by each of us in the course of a lifetime disappear by rolling or falling out of third dimensional space into the fourth dimension. If a Fourlander talked to us we would be as embarrassed as a Flatlander is should we talk to him. In traveling about, the Fourlander would use spheres for automobile wheels and in general would seem a superman to one of us. With such power it is easy to imagine a Fourlander becoming interested in the question whether higher space than four dimensions exists and so we might imagine him to go on. In fact, our algebra helps us to appreciate somewhat the sensibleness of such speculation.

Thus, by studying the various spaces, we are able to conceive a world of four dimensions. The unit in Pointland would be a point, that of Lineland a line segment, that of Flatland a square, and that of our land a cube. Such work stimulates the geometric intuition of the students, and incidentally puts him in possession of many important concepts.

To quote Professor Smith again:

"We agree that spaces of higher dimensions than the one in which we live can easily be conceived by analogy, and we agree without question to the paradoxes which we meet in the study of infinity, and yet we feel that it shows our great wisdom, or perhaps our boldness, if we deny the soul an existence. Strange, that in algebra we accept without the slightest question the idea of permanence of law, but that our little natures should so often boast that we deny the permanence of the soul! . . . We must face the inevitable, that in the domain of the infinite, the parts may indeed be equal to the whole, in spite of the childish beliefs of our finite minds. The mathematician continually meets this necessary conclu-

sion; it is a common-place; to him the laws of the finite give way without question when he enters the domain of the infinite. In all the finite, indeed, he sees the infinite. He knows that there is a one-to-one correspondence between the points in a sphere and those in the entire universe about him, and between the number of vibrating points in his brain and all the number in all space. In other words, within the brain of each of us, there is a point that corresponds to any given point in the universe, say one on the surface of Neptune. If we take another point, say the center of a fixed star, there is one special point, and one only, within each brain that also corresponds to that. If we move the point in the fixed star ever so slightly, we move the point in the brain so that it shall continue to correspond to it. Does all this signify anything to us? I certainly do not know. Does it mean that the planets have their influence upon us? I certainly do not know, but with my finite mind I am led to say I do not think so. Does this mathematico-physical relation of our brains to all the universe about us have any deeper significance? I do not know. But one thing I do know that thoughts like these give new meanings to the words, 'For, behold the Kingdom of God is within you'."[3]

The people who have studied the subject of the fourth dimension may be grouped as follows:

1. Those who are interested principally in investing the two- and four-dimensional people with all their queer antics and relations, as we have just done.

2. The mathematician who finds the language and ideas of the fourth dimension a help in drawing general conclusions. It is he who introduces the negative number, the notion of infinity, and the tangent of 90° for the purpose of enriching his vocabulary and attaining greater power of expression. Mathematics permits no exceptions.

3. The class led by Professor Zollner of Leipzig to whom the fourth dimension became the abode of the spirit world. For them it solved a great problem. The Bible is used as testimony and the extra dimension is read into the passage, "May have power to comprehend with all the saints what is the breadth and length and height and depth."[4]

In conclusion, it may be said that it is highly improbable that a fourth dimensional physical world really exists. As a mental conception it may have a place, but our physical experience includes only the three dimensional. A discussion or study of such a world does not do us any harm, and it does furnish us with thinking material that will sharpen our wits and furnish us with mathematical recreations.

All this is imagination, but it is interesting and it leads us to appreciate a term that is much used in astronomy at the present time—"fourth dimension." Whether such a dimension exists or not, the speculation sets us thinking more seriously of our position in the great universe about us. The possibility is perhaps no more remote than was telephoning across the Atlantic by wireless a few years ago, or sending pictures by radio, or crossing the ocean by airplane, or seeing at a long distance by the aid of electric waves.

[3] Smith, David Eugene, *op. cit.*, pp. 9–10.
[4] Ephesians 3: 18.

TODAY'S HIGH SCHOOL TO COLLEGE SITUATION IN MATHEMATICS

PHILIP PEAK
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This problem is one that probably needs several solutions. There appears to be several classifications of students and also several combinations of these classifications. From the high school to college we have one group of students who have excellent backgrounds in mathematics with a good elementary foundation and four years or more of high school preparation. A second group is composed of those who have four or more years of high school mathematics on their record but do not have the equivalent competency. A third group includes those who have very little or no high school mathematics. We find this group to be entirely too large.

After a student enters college he then reclassifies himself into one of several groups, the first may be the mathematics major in pure mathematics or applied mathematics. The second group may be those who need mathematics in order to use it in engineering, science and other fields. Members of the third group are those wishing mathematics for a general education or because of a personal interest.

The two sets of classifications, as you can see, may be paired in a number of ways and each one requires a special solution. I would like to present some possible solutions which may or may not be administratively possible. But first I'd like to point out some of the specific problems we have on the Indiana University campus.

Our requirements are such that the high school record in terms of units of mathematics determine the pattern of college work. If a student has one unit or less, he begins with a five-hour algebra course. If he has one and one-half units of algebra he begins with a three-hour algebra course. If he has two units of algebra he starts with trigonometry. If he has trigonometry or any course beyond that he omits that course in college. Students are not allowed to take any work that is equivalent to their high school work except they may audit such a course. With this system we find people with two years of high school algebra taking trigonometry and analytic geometry and not being able to do satisfactory work in it. The only work we seem to be able to accept from high school without question is trigonometry.

Another problem is that of shift in assignment technique. Often in college problems are not assigned, of if they are they may not be collected and returned corrected to the student. This causes the student who has never been required to evaluate his own work to flounder around in the dark. The average student also has difficulty

in working with the general case rather than specific examples. Much of the college work deals with the general case and requires a type of analysis the student has not been prepared to do. The student often finds himself dealing with problem material that he has no understanding of. The mathematics may not be difficult but he is not familiar with the physical laws or social situations on which the problem is based. Our students, and this applies to some of the better ones also, are extremely inaccurate in their arithmetical calculations. If these conditions existed with all students or with all students in a particular one of the classifications above the corrective procedure would be comparatively simple, but the conditions are found in various combinations.

Some of the things we have thought of doing within the college courses are as follows:

1. All people interested in mathematics would be placed in the program on the basis of a placement test irrespective of other factors.
2. To provide a non-credit course for all college students who do not meet a certain mathematics proficiency. Where the extra instructional staff could be recruited would be a serious problem.
3. There may be a need for a general course to provide mathematics for those who want only a general education.

Some changes in the high school that would appear to be beneficial are:

1. Better counseling to provide students with correct information concerning need for mathematics; college may need to help by distribution of information.
2. A consistent effort needs to be made so the student retains as much as possible of the four years work. This may mean some mathematics must be presented each year.
3. Perhaps the content of mathematics should be general each year rather than algebra one year, geometry the next, etc.

Communication between high school and college teachers:

1. Attendance at each other's professional meetings.
2. Conferences and workshops.
3. Syllabi for course work presented.

There are many problems related to all of these possibilities but the only solution seems to be that of continual effort with definite evaluation to determine the kind of progress that is being made.

Toll-road equipment, suitable for installation on turnpikes, uses photoelectric "eyes," weighing platforms and toll recorders to assess properly and quickly how much toll a given vehicle should pay. Its electronic "brain" also will figure traffic density to help officials utilize available manpower better. The device weighs vehicles and counts their axles as they roll toward the toll booth.

CONSERVATION EDUCATION: CRUCIBLE FOR ATTITUDE DEVELOPMENT

E. EUGENE IRISH
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The most casual inquiry into the field of conservation reveals the complexity of the problems involved and the diversity of opinions that are held. In fact, since its inception, the conservation movement has fairly bristled with "issues." Even those who might be termed "conservationists" have found controversies arising within their own ranks. Hammar suggests that "Much obvious confusion in the field of conservation is apparent because the conservationists have been unable to state clearly when a restrictive and when an expansionist or developmental policy should be followed."[1] Hammar is probably justified in his appraisal. But it should be pointed out that the confusion to which he refers stems largely from the social implications of some of the major issues of conservation. Perhaps part of the difficulty lies in the fact that in the past conservationists have not outlined objectives specific enough in nature to permit attainment. Neither have they agreed upon criteria or definitions of conservation in terms of specific resources. Ostrolenk says " . . . this is a difficult subject because there is no clear concept of what is meant by 'conservation' and by 'natural resources'."[2] Bunce conveys the same idea when he states that, "It appears impossible to define conservation in such a manner that it will apply with equal validity to all resources, unless it is done in such broad terms as to become practically meaningless. For this reason the use of specific definitions related to clearly defined cases seems desirable."[3] Zimmermann further substantiates the contention in the statement that, "Even if conservation is applied solely to natural resources such as water, soil, minerals, etc., it does not call for a single set of rules but for several, carefully adapted to the peculiar nature and requirements of the different types of resources."[4]

In light of the foregoing the schools may well ask, "What are we to teach about conservation?" or "If the conservationists themselves are not clear on some of these issues how can they expect us to be?"

Certainly a subject should not be excluded from the curriculum simply because of its controversial nature, although the distinguished Judd implied this in many of his writings. He ventured the opinion

[1] C. H. Hammar, "Society and Conservation," *Journal of Farm Economics*, XXIV (Feb., 1942), p. 109.
[2] Bernhard Ostrolenk, *Economic Geography*, Chicago: Richard D. Irwin, Inc., 1941, p. 763.
[3] Arthur C. Bunce, *Economics of Soil Conservation*, Ames, Iowa; Iowa Collegiate Press, Inc., 1945, p. 4.
[4] Erich W. Zimmermann, *World Resources and Industries*, New York: Harper and Brothers, 1933, p. 796.

that it was less harmful to exclude the discussion of public utilities than to suffer the pro-trust propaganda in the schools![5] As for the question of what to teach in conservation Burnett puts it this way: "The question is not to teach 'fundementals,' but rather what information, attitudes, and skills *are* fundamental for optimum living in America today."[6]

Too often teachers assume that if students acquire certain factual information they will behave in certain desirable ways. Mere knowledge of a particular referent,* however, does not preclude favorable behavior in respect to it. Witness the incident related by Palmer about the biology teacher who " . . . taught a lesson on fish by using undersized trout that he captured in a net out of season, in a posted area, and that he held captive without a licence to do so."[7] This example, together with many others that might be cited, serves to emphasize the fact that behavior is contingent upon other factors besides knowledge. There is considerable evidence to indicate that one of the most potent of these factors is attitude. Young puts it rather succinctly by stating that, "All we can say is that when the time comes to act the attitude will enter in as an essential factor."[8]

If attitudes are the determinants of behavior, as Hoover[9] has pointed out, what are the implications for teachers of science, especially when they are dealing with a course or a unit as personally and socially significant as is conservation? What attitudes are the desirable ones? What source materials should be employed? Should the teacher make known his own opinions, and if so, at what point during the instruction? These and many other questions will legitimately intrude upon the minds of thoughtful teachers. There will be some who will even doubt that attitudes are teachable. Not too long ago a scientist, distinguished in his own field of electro-chemistry, stubbornly maintained that attitudes can *not* be taught! To such people one can only cite the evidence of experimental research and hope for the best. Thorndike has summed up the results of the research in these words:

We now know that the fundamental forces which can change desires and emotions, directing them into desirable channels, are the same as change ideas and

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[5] Charles H. Judd, "Scientific Techniques of Curriculum Making," *School and Society*, XV (Jan. 7, 1922), pp. 1–11; "Curriculum: A Paramount Issue," N.E.A., *Proceedings*, 1925, pp. 805–811; "Method of Securing National Educational Standards," *Educational Record*, IX (April, 1928), pp. 81–95; "Place of Research in a Program of Curriculum Development," *Journal of Educational Research*, XVII (May, 1928), pp. 313–323.

[6] R. Will Burnett, "Conservation: Focus or Incident in Science Education," *Science Education*, XXVIII (March, 1944), p. 84.

* A referent is here defined as anything—a person, a group of persons, an institution, an object, or an idea.

[7] E. Lawrence Palmer, "Conservation Education in the Schools; Report of School Activities and Suggestions as to Programs," *Nature Magazine*, XXXII (Nov., 1939), p. 512.

[8] Kimball Young and Others, *Social Attitudes*, New York: Henry Holt and Co., 1931, pp. 8–9.

[9] Floyd W. Hoover, "Attitudes, Those Elusive-Behavior Determinants," *Educational Administration and and Supervision*, XXXI (April, 1945), pp. 215–222.

actions. A human being learns to react to the situations of life by such and such wants, interests, and attitudes, as he learns to react to them by such and such percepts, ideas, and movements. In both cases, the task of education is to cause the desired connection to occur and to attach the confirming reaction to it.[10]

That attitudes can be taught is implicit in the objectives set forth by some of the official committees on science teaching.[11] If they can not be taught the listing of them as important objectives would appear to be a rather dubious procedure.

With respect to the changing of attitudes teachers of conservation are referred to the study by Williamson and Remmers.[12] They set out to:

(1) measure persistence of group attitudes changed by defined social stimulus material,
(2) measure the respective variabilities of the groups to discover whether they become more or less homogeneous after the presentation of such material,
(3) compare rural with urban group-attitudes produced by defined social stimulus material and to compare their relative homogeneity after the presentation of such material.

The five attitude objects were:

(1) allowing the government to tell the farmer how to farm,
(2) allowing each farmer to farm as he pleases,
(3) clean farming,
(4) taxing all the people to plant new forests, and
(5) draining swamps.[13]

In their results the investigators have this to say: " . . . with exposure to information about an issue, stereotyped attitudes appear to break down and become individualized."[14] The conclusions reached are deemed to be of sufficient interest to justify their whole inclusion:

(1) the attitudes of high school pupils toward certain conservation issues can be significantly changed in a desired direction.
(2) the attitudes of high school pupils toward certain conservation issues, having been changed by defined stimulus material, tend to persist as changed after a lapse of as much as eight months.
(3) the attitudes of the group were generally less homogeneous after presentation of the stimulus material than before.
(4) the rural group tends to be less affected on the average by the stimulus material than the urban group.[15]

A close examination of the techniques employed in the experiment just cited raises the question of what is meant by "defined social

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[10] Edward L. Thorndike, *The Psychology of Wants, Interests, and Attitudes*, New York: D. Appleton-Century Co., Inc., 1935, p. 217.

[11] Annual Report of the Conservation Committee of the Central Association of Science and Mathematics Teachers, SCHOOL SCIENCE AND MATHEMATICS, XL (Jan., 1940), p. 74; *Science Education in American Schools*. Forty-sixth Yearbook of the National Society for the Study of Education, Part I, Chap. III, Chicago: Distributed by the University of Chicago Press, 1947.

[12] A. E. Williamson and H. H. Remmers, "Persistence of Attitudes Concerning Conservation Issues," *Journal of Experimental Education*, VIII (March, 1940), pp. 354–361.

[13] *Ibid.*, p. 354.

[14] *Ibid.*

[15] *Ibid.*, p. 361.

stimulus material." It might be argued that any attitude whatsoever can be induced by the presentation of suitably "defined social stimulus material." Another question that arises, and one closely related to the first, is what was the nature of the teaching procedures employed. The answers to these two questions may lend a clue to the questions formerly raised in regard to what attitudes are desirable and whether or not the teacher should make known his own opinions. As for the latter, an appropriate reply is suggested in the following paragraph:

> Respect for pupil personality requires that a teacher will refrain from presenting a single solution to a pupil as a sort of fixed pattern that a pupil must accept. The democratic and American way of solving a social problem demands that the greatest number possible of our future citizens shall be taught to consider fairly the arguments for each of a number of alternative solutions. Hence there can be no question as to the teacher's right to deal with controversial issues, if the continued improvement of our fundamental institutions is to be assured. But to be able to "get away with this" the teacher must have a very broad scholarship, the ability to deal with delicate matters in an impersonal, objective, and fairminded way; the disposition to keep an argument free from emotion; and the habit of keeping his own views out of the picture until some pupil says, "Where do you stand?" then taking a definite position, but admitting that we all have our biases and prejudices and that at best his own opinion is merely one of alternative solutions.[16]

Any attempt to answer the first question must be predicated upon the method employed to achieve the desired attitudes. Two such methods are possible in the establishment of new values:

1. *propaganda* in which we (the powers that be) decide what values should be set up, and which resorts to any technique to accomplish its ends,
2. *education* which permits the learner to decide what values are the most efficacious, and which implies much greater respect for the individual.

If it is agreed among teachers that education shall be the way, the answer as to what attitudes are desirable is *scientific attitudes*.

It may be well to remind those who worship pure science to the extent of slavish adherence that " . . . all ideas, being social in origin, have, in a certain sense, social implications; and sometimes the social implications of what appear to be purely individual or abstract ideas are even more important than specific attitudes toward social relationships and problems."[17] "Science is not a technique or a body of knowledge, though it uses both. It is rather an attitude of inquiry, of observation and reasoning with respect to the world. It can be developed, not by memorizing facts or juggling formulas to get an answer, but only by actual practice of scientific observation and reasoning."[18]

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[16] Raleigh Schorling and Howard Y. McClusky, *Education and Social Trends*, New York: World Book Company, 1936, p. 140.

[17] Merle Curti, *The Social Ideas of American Educators*, New York: Scribner's Sons, 1935, p. xiv.

[18] Karl T. Compton, "Science in Education," *Science Education*, XX (April, 1936), pp. 53–56.

The broad field of conservation, which demands a thorough understanding of the complex interrelationships of the social and economic forces with the physical, provides excellent opportunities for teachers to stage learning experiences, from which scientific attitudes may be derived. It is, indeed, a crucible for attitude development.

THE MASTER'S DEGREE IN PHYSICS AT MICHIGAN STATE

Michigan State College will take the lead this year in offering a new program for obtaining the master's degree in physics—entirely through summer session work.

Students or teachers with bachelor's degrees in physics, or equivalent training, may complete requirements for the M.S. degree by attending four successive nine-week summer sessions, according to Dr. Thomas H. Osgood, Dean of the School of Graduate Studies.

The new program, which is in addition to graduate physics work available during the regular academic year, will offer graduate and advanced undergraduate courses. Most of these may be taken for graduate credit.

Beginning June 22 this year, advanced undergraduate courses will be offered in mechanics, electronics, modern physics, and radioactivity. These and advanced undergraduate courses in electricity and magnetism and in physical optics will be offered at least once during four successive summers.

Graduate courses, Dean Osgood said, will include introductory theoretical physics (mechanics), nuclear physics, electromagnetic theory, thermodynamics-statistical mechanics, atomic and molecular spectra. Each of these will be offered every other summer, with the exception of thermodynamics. It will be offered at least once every four years.

The four-year program is expected to be especially helpful to teachers, because of the "summer-only" work. Also interested are students, and workers in research laboratories, engineering firms and industrial organizations.

The 1954 summer session catalog may be obtained from the Director of the Summer School, Michigan State College, East Lansing, Mich. The Graduate School catalog and information about admission to the School of Graduate Studies may be obtained from the Dean of the School of Graduate Studies.

NAUTILUS POWERED BY FIRST ATOMIC ENGINE EVER TO PROPEL A SHIP

The U.S.S. *Nautilus*, will be powered by the first atomic engine ever to propel a ship.

Built by Westinghouse Electric Corporation, under contract with the Atomic Energy Commission, the *Nautilus*' atomic power plant actually ranks as the second full-size power-producing atomic engine in the nation's history—the first (Mark I) being a land-based prototype.

But the *Nautilus* engine (Mark II) will be the first ever to provide the motive power for any vehicle or vessel by controlled nuclear fission. With most components of the engine now installed inside the *Nautilus* hull, Mark II is the most powerful submarine engine ever built. Fleet type submarines in World War II had engines of about 6,000 horsepower.

Capable of speeds above 20 knots while submerged, this atomic engine brings to reality the dream of useful power extracted from the same material—uranium 235—that was the heart of the first atomic bomb.

Because a nuclear reactor of this type does not require oxygen for its operation —as does a combustion engine—the *Nautilus* will be able to operate at top efficiency for long periods of time while submerged. It thus will be the first true submarine.

MORE MATHEMATICAL LINGERING: A MAXIMUM THEOREM

LESTER H. LANGE
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In a former issue of this JOURNAL* I gave a concrete example of the worthwhile practice of reflecting on a mathematical problem even after we have succeeded in solving it. It was shown that this habit can yield the discovery of deeper, more inclusive theorems. The rather inclusive theorem of this present paper will be shown to have been discovered in much the same spirit and other subsequent investigations will be suggested.

Perhaps you have heard of this problem, which I shall call:

PROBLEM A: Prove that the largest rectangle which can be inscribed in a given right triangle has the same area no matter on which side of the triangle the base of the rectangle lies.

Now, in order to solve a problem, we must begin in some way; we must get hold of it in some fashion. I think it would not be unnatural to arrive at the consideration of:

PROBLEM B: The altitude of a triangle is a and its base is b. Find the dimensions of the rectangle with maximum area that can be inscribed in the triangle with one side along the base. See figure 1.

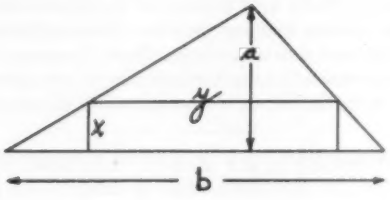

FIG. 1

This problem may be stated:

Given: a, b

Find: x, y

Condition: $x \cdot y =$ Maximum.

From similar triangles it is seen that

$$\frac{a-x}{a} = \frac{y}{b}, \tag{1}$$

and hence

* Lester H. Lange, "In Mathematics Too: Linger And Learn," June, 1953, pp. 478–483. In this article several alternative and perhaps successively more elegant proofs were discovered in a problem involving congruences connected with a "calendar problem."

$$x \cdot y = (x)\left(\frac{b}{a} \cdot \frac{a-x}{}\right) = \frac{b}{a}(x)(a-x).$$

To find the maximum of (xy) it clearly suffices to find the maximum of the product $(x)(a-x)$.† We can proceed to find this maximum in the following way:

$$(x)(a-x) = -x^2 + ax$$
$$= -\left(x - \frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2$$

and the expression on the right clearly has a maximum when

$$x - \frac{a}{2} = 0, \quad \textit{i.e.}, \quad \text{when } x = \frac{a}{2}.$$

Hence, the maximum of (xy) is given by

$$\frac{b}{a}(x)(a-x) = \left(\frac{b}{a}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right) = \frac{ab}{4}$$

and hence $y = b/2$ for the maximum rectangle. The answer to our present problem then is,

"The dimensions are $x = \frac{a}{2}$, $y = \frac{b}{2}$."

But let's stay with this problem a minute. Look at the form of the answer for the maximum area. It says that the maximum rectangle has area

$$A_m = \frac{1}{2}\left(\frac{ab}{2}\right) = \frac{1}{2}(\text{area of given triangle}).$$

Now, the "given triangle" in this problem (PROBLEM B) is specified only with respect to a base and its area, not really with respect to its shape; for the triangle need only be of a shape which is such that the vertex opposite the given base lies on a line parallel to that base and at a distance a from it. *Our way of solving* the given problem requires that the vertex in question be "above" the given base, *i.e.*, that a base angle of the triangle must not be greater than a right angle; for equation (1) is no longer valid if a base angle is greater than a right angle. In fact, we have not solved PROBLEM (B) completely; we have a *restricted* solution of the problem. We must derive a more general solution. This I will do in a moment, but first I shall apply

† G. Cantor, in his *Geschichte der Mathematik*, Vol. I, p. 228, says that the solution of this problem in Euclid's *Elements*, VI, 27, is the first example of a maximum in the history of Mathematics.

the results obtained thus far to the problem first stated, PROBLEM A.

What we have just proved can be written as:

THEOREM I: The maximum area of a rectangle inscribed in a given triangle is equal to half the area of the given triangle, provided that a) one side of the rectangle lies along a side of the triangle, and that b) *this* side of the triangle does not have an obtuse angle of the triangle at one of its extremities.

We need now make only an elementary observation and find that the simple analysis of our partial solution to PROBLEM B has led to an elegant solution of PROBLEM A, *viz.*, a *right* triangle has *no* obtuse angles and THEOREM I disposes of PROBLEM A. And, incidentally, our theorem establishes a more general result than is inherent in the conclusion of PROBLEM A: we can now see very clearly what rôle is played by the fact that a *right* triangle is specified. We might even state this observation as:

THEROEM II: The maximum area of a rectangle inscribed in a triangle which has *no obtuse angles* is equal to half the area of the given triangle, provided that a side of the rectangle lies along a side of the triangle. (In one way, this is a more general theorem than the previous one; in another sense, it is a more restricted one.)

There now remains the necessity of seeking a less restricted solution to PROBLEM B. What if there *is* an obtuse base angle in the triangle we consider? See figure 2. Now, in some way, the *shape* of

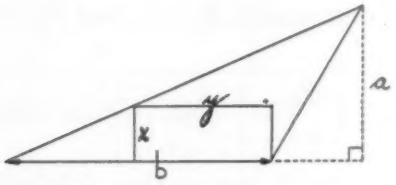

FIG. 2

the given triangle must be specified. We, quite naturally, consider the quantity z in figure 3. Once again employing similar triangles,

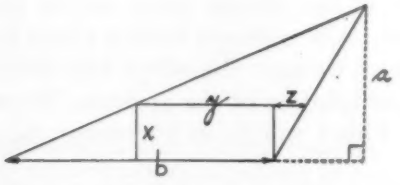

FIG. 3

we have

$$\frac{a-x}{y+z}=\frac{a}{b},$$

or

$$y=\frac{b}{a}(a-x)-z. \tag{2}$$

The difficulty here is that y is no longer a function of x alone—for in this instance, the more complicated shape of the triangle induces a complication in the nature of y. But once the shape of the triangle is given, along with the numbers a and b, the number z, while not a constant, is certainly determined for any choice of x. In fact the ratio z/x is a constant. We introduce the quantity p, which will serve as the number needed to tell us the shape of the given triangle. (See figure 4).

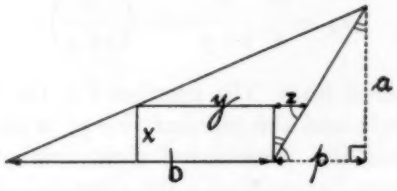

FIG. 4

By similar triangles, we see that $z=(p/a)(x)$. This result we substitute into equation (2) and find that

$$ay=ab-(b+p)(x). \tag{3}$$

We wish to maximize the product (xy) and therefore consider

$$\frac{a}{b}(xy)=(x)\left(a-\left[\frac{b+p}{b}\right]x\right),$$

or

$$\frac{a}{b+p}(xy)=(x)\left(\frac{ab}{b+p}-x\right).\ddagger$$

Now, similar to our earlier observation, the maximum of (xy) will be found if we find the maximum of $[a/(b+p)](xy)$. We could find this maximum with the same technique employed earlier and, of course, arrive at the result: when (xy) is a maximum, the value of x is given by

‡ A check. If $p=0$, this reduces to $(x)(a-x)$ as before.

$$x=\frac{1}{2}\cdot\frac{ab}{b+p}\cdot\S$$

Also, from (3),

$$ay=ab-(b+p)\left(\frac{1}{2}\cdot\frac{ab}{b+p}\right),$$

and hence, $y=b/2$. (Note that this value of y is independent of the number p. Does this make sense?) The area of the maximum rectangle is

$$A_m'=\frac{1}{4}\ \frac{ab^2}{b+p},$$

which, we notice with satisfaction, reduces to our former result when we set $p=0$. This result may be written as

$$A_m'=\frac{ab}{4}\ \frac{b}{b+p}=A_m\left(\frac{b}{b+p}\right),$$

an easily remembered form. The number b is the length of the base or the given triangle and the number $(b+p)$ is the base of what we may choose to call the "associated right triangle." (To be more specific, let the term "associated right triangle" designate the right triangle which exists when the vertex opposite the given base is *not* "above" the base, *i.e.*, $p>0$. We have already disposed of the case where there was no obtuse base angle.) This terminology now enables us to state a quite general theorem:

THEOREM III: The maximum area of a rectangle inscribed in a given triangle—one side of the rectangle lying along a specified base of the given triangle—is equal to half the area of the given triangle multiplied by the ratio of the base of the triangle to the base of the associated right triangle; this ratio being taken as unity when the associated triangle does not exist.

We might write this compactly as

$$A_M=\left(\frac{b}{b'}\right)\frac{A}{2},$$

where

A_M = Area of maximum rectangle
A = Area of given triangle

§ This result can, of course, be found in a number of other ways. For example: An expression of the form $(x)(c-x)$ can be graphed, yielding a parabola with axis parallel to the vertical axis and crossing the X-axis at $x=0$ and at $x=c$. The maximum point of the parabola has an abscissa half-way between the values 0 and c.

b = Base of given triangle
b' = Base of associated right triangle.

This is certainly an easily remembered form for our results. Furthermore, the maximum rectangle is easy to construct in any case, for the base of the maximum rectangle will always be half the base of the given triangle.

Finally, other investigations suggest themselves. For example:

a) Are there cases where a larger rectangle can be placed in a triangle when we do not require that a side of the rectangle lie along a side of the given triangle?
b) How does the area of the inscribed circle of the triangle compare with these results?
c) Can we discover an even more general theorem if we do not require that a side of the "inscribed" rectangle be entirely *included* in the given base of the triangle? What if rectangles like the one shown in figure 5 are considered?

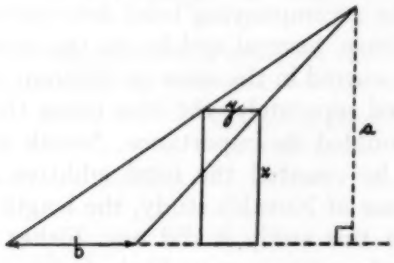

FIG. 5

Clearly, equation (1) applies and hence the properly modified conclusion applies.

OAK RIDGE INSTITUTE OF NUCLEAR STUDIES

The Special Training Division of the Oak Ridge Institute of Nuclear Studies, Oak Ridge, Tennessee, has scheduled a two-week summer course on radioactivity, designed for teachers of science education and secondary school science.

Entitled "Special Course in the Use of Radioisotopes in Secondary School Science Teaching," the course will begin on June 7, and will consist of background lectures in physics and radioactivity. Also included in the curriculum will be demonstrations of the experiments outlined in the recent Atomic Energy Commission booklet, *Laboratory Experiments with Radioisotopes*, by Samuel Schenberg and John H. Harley. This booklet is based on a science education program developed by the Board of Education of the City of New York.

Emphasis throughout the course will be on the technique of using radioisotopes in classroom demonstrations and in laboratory work.

Further information about this course, and application blanks, may be obtained by writing the Special Training Division, Oak Ridge Institute of Nuclear Studies, P.O. Box 117, Oak Ridge, Tennessee.

SCIENCE IN THE *NEW YORK TIMES*

CATHARINE BERGEN
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To examine science articles in the *New York Times* is not a new undertaking. A study of the *Times* for several years during the 1930's was made by Novak* more than a decade ago. Other studies have been published at least as long ago as 1923. This study is based upon the Annual *New York Times Index* which had its origin in the year 1930. Using this index limits the investigator to an examination of topics and numbers of articles. It thus prevents a direct comparison with Novak who measured column inches rather than number of articles.

SCIENCE DURING 1950

The year 1950 was singled out for special analysis. The author went through the entire *Index* for that year noting all articles which from their titles and the accompanying brief description, seemed to deal primarily with science. Several articles on the same topic regardless of whether they occurred in the same or different issues of the newspaper were counted separately, the idea being that two articles on the same topic doubled its importance. Novak did essentially the same thing since he counted the total additive lengths of all the columns. In the case of Novak's study, the length of a single article carried weight. In this study it did not. Either technique can be justified. A ten inch article is quite likely to be of more significance than a five inch one but not necessarily twice as much so. The inclusion of the topic in the *Times* is independent of the complexity of the subject, but the length of the articles is to some extent dependent upon that very complexity.

Articles dealing primarily with commercial or legal aspects of science or notices of science meetings were omitted in this part of the study. As usual these were borderline cases where it was a matter of opinion as to whether an article should be counted or not. The *New York Times Index* is organized with numerous headings and subheadings. In this part of the study the author used her own judgment in classifying the articles and did not necessarily follow the headings of the *Index*.

For 1950 the present author found a total of 999 articles dealing primarily with science. These have been classified by her as shown in Table 1. The column concerning "Highlights of 1950" contains science articles included in a special section by that title which precedes

* Novak, Benjamin J. *An Analysis of the Science Content of the New York Times and of Selected General Science Textbooks*. Philadelphia, 1942.

the regular listing in the *New York Times Index*. It can be seen that physiology is the most frequent subject and when combined with the related biochemistry constitutes nearly 40% of all articles. Novak found that the topic which he called "health and medicine" contained about 20% of the science space. The discrepancy may come from the different criterion of measurement used by Novak, from a sporadic variation from year to year, or very likely from the fact that Novak included articles on gardening in his study. The present investigator included scientific research in agriculture but did not include what is ordinarily referred to as gardening. There is not sufficient evidence to conclude that interest in physiology is increasing.

When articles about zoology are interpreted to include descriptions of animals or their behavior, birds, zoos, etc., they constitute over 10% of the total. Another 10% roughly belong to the area of physics including atomic energy (the *Times* has classified atomic energy separately from physics). All other topics occur in less than 10% of the total articles. The percentages are somewhat different for the

TABLE 1. DISTRIBUTION OF SCIENCE TOPICS IN THE *New York Times*

Topics	Number of articles in 1950 *Index*		Number of articles in "Highlights of 1950"	
Aeronautics	50			
Air pollution	42			
Anthropology and Archaeology	10		4	
Astronomy	58		1	
Botany and Agriculture	57			
Chemistry and Products	71		1	
Biochemistry	38	396	4	10
Physiology	358		6	
Geology	5		1	
Oceanography	8			
Physics	36	109	1	8
Atomic energy and Nuclear Study	73		7	
Meteorology	31			
Photography	28			
Psychology	8			
Zoology (incl. animals, birds, zoos, etc.)	126			
Totals	999		25	

"Highlights of 1950" and are, of course, much less reliable because of the small total of 25 science articles. It is interesting to note that here, too, however, physiology and biochemistry together make the largest single group; and that physics with atomic energy rank a close second. Zoology has definitely lost out in the "Highlights" and archaeology has become prominent. The 4 archaeology articles in-

clude: recent finds in Iraq which may change the world's cultural history, the discovery of a murder trial record from Sumerian tablets, the finding of St. Peter's tomb in Rome, and the radiocarbon assay of relics.

Atomic and Nuclear Science

In view of the role played by the atomic bomb during and since the last war, it seemed desirable to make a special study of this topic using the annual volumes of the *New York Times Index* from its inception in 1930 to date. (The annual volumes began in 1930—not the *Index* itself which is even today available at more frequent intervals.) For this part of the study the author utilized only those articles which the *Index* classified under headings such as atomic energy or nuclear research. During some years nuclear research was a subheading under physics, later it became a separate heading of its own. Atomic energy is used as a heading for more than science articles. Since World War II it has contained hundreds of articles but most of these deal with legal, defense, economic, or international control aspects of atomic energy and very few with science. Those which do deal with science, however, are included with the articles classified under nuclear research. Before 1945 atomic energy was also a main heading in the *Index* but nearly all articles listed in those years deal with the scientific aspects. The actual count of atomic energy articles can be seen in Table 2. The count was not made for 1945 or 1946 since for these 2 years the science articles were buried amongst so many others listed as atomic energy. In 1947 and later a research sub-head under atomic energy made the counting more feasible. Since the total number of atomic energy articles in 1945 and 1946 each was comparable to 1947, it is possible that the number of science articles included might be somewhat similar as well. (It can be seen that the number of articles for 1950 according to Table 2 is 60. In Table 1 for 1950 the number of articles on atomic and nuclear physics is stated as 73. The lower figure in Table 2 results from the technique of utilizing the *Times Index* headings and not looking through all headings. Atomic research listed as chemistry or biological subjects would, for example, have been included in Table 1 but missed in Table 2. All figures in Table 2 will tend to be too low. These two values for 1950 give an index of the reliability of the technique used in the second part of the study. To have used the more thorough technique for all the years involved would have taken several hundred hours of tedious labor and probably not have been justified.)

The effect of the war on these articles cannot be missed. Through 1941 the variation appears to be sporadic. In 1942, 1943, and 1944 we can see the effect of the policy of secrecy. Those few articles dur-

TABLE 2. *New York Times* ARTICLES ON ATOMIC AND NUCLEAR PHYSICS

Year	Number of articles	Year	Number of articles
1930	29	1942	12
1931	32	1943	3
1932	55	1944	1
1933	52	1945	*
1934	48	1946	*
1935	28	1947	75*
1936	63	1948	54*
1937	38	1949	37*
1938	26	1950	60*
1939	39	1951	58*
1940	38	1952	53*
1941	32		

* From 1945 hundreds of articles are classified under *atomic energy*. Most of these deal with social science aspects of the subject and are not included here. See text for clarification.

ing these years did not deal with reactions related to the atomic bomb. Directly following the war there was evidently a rush to bring the public up to date on the subject. At present the frequency of articles is settling back to not much more than the pre-war level. These are the articles about theories and research, of course. Articles on social science aspects of atomic energy are still running probably well over a thousand a year.

GENERAL ARTICLES ON SCIENCE

Under the heading of science, itself, the *New York Times Index* includes articles and editorials of general interest. Discussions of the importance of science, its impact on society or religion, its importance to industry, or its general development are found here. The

TABLE 3. GENERAL ARTICLES ON SCIENCE IN THE *New York Times*

Year	Number of articles	Year	Number of articles
1930	51	1942	45
1931	42	1943	53
1932	37	1944	62
1933	37	1945	281
1934	24	1946	292
1935	30	1947	175
1936	89	1948	114
1937	56	1949	111
1938	71	1950	122
1939	65	1951	90
1940	46	1952	100
1941	57		

articles under this classification were counted and found to be as shown in Table 3. Here again the variation before the war is probably without significance. There is no change during 1942, 1943, and 1944 as there was in the case of atomic energy. Obviously secrecy did not apply to science as a whole. The effect of the war on the frequency of such articles from 1945 on, however, is tremendous. The number of articles is increased five or six times. This is followed by a gradual decrease until 1948 where a new plateau seems to have started. This plateau has not returned to the pre-war level as with the atomic and nuclear research articles but remains something like twice as high. No doubt it was the atomic bomb which acted as a stimulant. Whether or not the new plateau will hold indefinitely remains to be seen. It has held for 5 years after the abnormally high figures for the 3 years before that.

RIDER RELEASES TWO BOOKS ON COLOR TELEVISION

Two new books on Color Television fundamentals have been released by John F. Rider Publisher, Inc., 480 Canal Street, New York 13, N. Y. One of them titled *Highlights of Color Television* is written by John R. Locke, Jr., a color TV expert in the employ of the General Electric Co. This forty-eight page publication is a "quickie" dealing with the highlights of the N.T.S.C. system. The author explains colorimetry, the N.T.S.C. color signal, the organization of the transmitting system, the color receiver including the color bar test pattern, matrixing, color sub-carrier generation and synchronous detection in a very easy-to-understand style. The book is $5\frac{1}{2} \times 8\frac{1}{2}$ inches, twenty-nine illustrations, paper covered and sells for $.99.

The second book titled *Introduction to Color TV* is written by Milton Kaufman of the Color Television Laboratory of CBS-Columbia and by Harry Thomas a member of the research staff of the Federal Telecommunications Laboratory, both of whom are extremely familiar with the subject of color television. This book is the complete story on the subject. The authors cover all aspects of color television beginning with answers to questions which the public may ask and going right through the organization of the N.T.S.C. color transmitting system and color receiving system, ending up with descriptions of the adjustments on color receivers.

The authors have taken special pains to explain without the use of mathematics such details as the different color processing circuits in receivers, the I and Q signals and color difference signals. Special effort is made to present the functioning of matrixing and synchronous detection in detail so that it is completely understandable without the use of mathematics. Two schematics of different color television receivers are included. *Introduction to Color TV* is written in such style that the subject is completely understandable by all who have any familiarity with black and white television. 140 pages, $5\frac{1}{2} \times 8\frac{1}{2}$ inches, 74 illustrations, paper covered and sells for $2.10.

Chemical weeder, when first applied, kills weed seeds then undergoes a rapid chemical change to become a highly effective fertilizer. The chemical, calcium cyanamide, is said to supply soil-sweetening lime and plant-feeding nitrogen during the entire growing season.

AN INTERESTING EQUATION: Tan $2X = \text{Cot } X$

RAY JURGENSEN
Culver Military Academy, Culver, Indiana

The equation proves interesting when we consider the inadequacy of

Solution A

$$\frac{2 \tan x}{1-\tan^2 x} = \frac{1}{\tan x}$$

$$3 \tan^2 x = 1$$

$$X = 30°, 150°, 210°, 330° \quad (\text{for } 0° \leqq x < 360°).$$

Solution A seems natural and innocent enough; it seems to involve no extraordinary trigonometric or algebraic techniques.

But suppose we attack $\tan 2x = \cot x$ in various apparently less efficient ways. Proceeding from

$$\frac{\sin 2x}{\cos 2x} = \frac{\cos x}{\sin x},$$

or from

$$\tan 2x = \sqrt{\frac{1+\cos 2x}{1-\cos 2x}},$$

or from $1/\cot 2x = \cot x$, we obtain not only the roots given in Solution A but the roots 90° and 270° as well. Graphs of $\tan 2x$ and of $\cot x$ on the same pair of axes reveal that 90° and 270° are roots. In fact an inspection of the original equation yields the roots 90° and 270°.

Where does the weakness of Solution A lie? Can't we have confidence in completeness of solutions when trigonometric identities are used? We soon observe that, for $x = 90°$, only finite terms are found in the original equation whereas $\tan 90°$ is found several times in the first step of Solution A. A graph of the function

$$f(x) = \frac{2 \tan x}{1-\tan^2 x} - \frac{1}{\tan x}$$

reveals that as x approaches 90° or 270° the function approaches zero in value. (When $x = 89°$, $f(x) = -.05238$; when $x = 89°30'$, $f(x) = -.02619$; when $x = 89°59'$, $f(x) = -.00087$.) The weakness of Solution A lies in superficial treatment of the first step. We should be alert to the possibility of

$$\frac{2\tan x}{1-\tan^2 x} \quad \text{and} \quad \frac{1}{\tan x}$$

approaching zero simultaneously. That is, we should consider the possibility of the denominator of

$$\frac{2}{\frac{1}{\tan x}-\tan x}$$

and the denominator of $1/\tan x$ becoming infinite at the same time. Then we would obtain the roots 90° and 270° readily enough.

Similar precautions must of course be observed in other solutions involving functions which take on infinite values. One striking example is the equation $\tan 16x=0$, which has thirty-two non-negative roots less than 360°. Only two roots survive if the identity

$$\tan 2A=\frac{1}{1-\tan^2 A}$$

and the technique of clearing fractions are both employed four times in mechanical fashion! Even worse, a careless worker can proceed from

$$\tan 16x=0 \quad \text{to} \quad \frac{1}{\cot 16x}=0$$

to the embarrassing result $1=0$.

THE TEACHING OF MATHEMATICS

WILLIS B. CATON
De Paul University, 64 East Lake Street, Chicago 1, Illinois

The observations which I shall make are neither novel nor new but appear to be worth a few moments of examination. I should like to propose that somewhat more emphasis be placed on the fundamental ideas and concepts all along the line from the college level down through the high school and into the elementary school levels. I realize that this will be difficult to do, even at the college level, and much more so at the high school and elementary school levels. However, it would appear to be worth trying, as it seems reasonable to expect that students would do better if they have clearer notions about the basic concepts.

Thus, one would start early in the grades to develop the student's

concept of natural number, positional notation, addition and multiplication with a detailed discussion of such processes as "carrying." This would mean more time spent on the "whys and wherefores" at the expense of the purely computational work, but would perhaps be more valuable to the student in the long run.

At the beginning of the subject of rational numbers, there would again be detailed discussions, with numerous illustrations of various fractional parts of pies, etc. It would appear that a clear understanding of these ideas should make for a more rapid development of the computational facilities of the student.

Next, the concept of positive real number could be introduced as an infinite decimal. We know that this is the most difficult step of all and will require the greatest patience and effort on the part of the teacher.

I would not attempt to say at what point in the student's development, these various ideas should be presented, nor would I prescribe the methods of presentation. These questions should be left to the experts in the field of elementary education. However, I do believe that some such treatment as that found in Chapter III of Richardson's "Fundamentals of Mathematics" could be used as a guide in a development of this subject at the elementary school level.

In elementary high school algebra, the new ideas of negative numbers, literal expressions, equations, complex numbers, graphing and the concept of function, should be taught in as nearly a logical fashion as possible. Each of these presents a hurdle to the student and will require ample time for assimulation and digestion. Of course, those students who take trigonometry will have a second opportunity to try to understand the function concept.

In high school geometry, the teacher has a splendid opportunity to try to get across to the student, the important concept of a mathematical system with its postulates and theorems deduced therefrom. An excellent guide for this is Hilbert's "Foundations of Geometry." Also the book "Basic Geometry" by G. D. Birkhoff and R. Beatley is excellent. The teaching of mathematics as a deductive science, is in itself quite a chore. For, as the geometry teachers know, the student will object violently to proving theorems whose obviousness is apparent from a suitable figure. However, if the student can be pulled up the hill, if he can be made to understand the "spirit of the game," the teacher has really performed a major accomplishment. Here in geometry, in connection with length, area, and volume, the student will come to grips with the notion of limit.

If the student goes onto college, his algebra and trigonometry, will be somewhat of a review with additional refinements and elaborations. If he takes analytic geometry, he will have ample

opportunity to use all knowledge acquired to date, and in addition to handle geometric problems by means of algebraic tools. If he survives all this and decides to take calculus, he begins all over again with a somewhat more mature formulation of each of the concepts of number, function, and limit. Here again, the teacher must walk a tight rope, with the ideas and basic concepts on one side, formal manipulations and applications on the other. An excellent source of material on the calculus is to be found in the two volumes by Richard Courant.

In our work-a-day world of presenting abstract ideas, our accomplishments often seem meager. We must continue to hammer away along the same lines for, our goal is a high one. Many times, at the end of a calculus course, students have admitted that they are still in the dark about the idea of limits; but on the other hand, some have understood, and for these we are thankful.

PROBLEM DEPARTMENT

CONDUCTED BY G. H. JAMISON
State Teachers College, Kirksville, Mo.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and to solve problems here propsoed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution, or proposed problem, sent to the Editor should have the author's name introducing the problem or solution *as on the following pages.*

The editor of the department desires to serve its readers by making it interesting and helpful to them. Address suggestions and problems to G. N. Jamison, State Teachers College, Kirksville, Missouri.

THE EDITOR'S ADIEU

May I as retiring editor say "thank you and good-bye" to a host of friends made through this department during these happy years. I wish I could pay tribute by naming many of you who have by your contributions made this department possible. I feel that I know many of you. Some of you I have met. Thanks and good-bye.

G. H. JAMISON

SOLUTIONS AND PROBLEMS

Note. Persons sending in solutions and submitting problems for solutions should observe the following instructions.

1. Drawings on India ink should be on a separte page from the solution.

2. Give the solution to the problem which you propose if you have one, and also the source and any known references to it.

3. In general when several solutions are correct, the ones submitted in the best form will be used.

LATE SOLUTIONS

2389. *Charles Bibeaa, Biloxi, Miss.; Nathaniel Grossman, Aurora, Ill.*

2393, 4. *Norman Anning, Alhambra, Calif.*

2387. *J. W. Lindley, Amarillo, Texas.*

2391. *J. B. King, Denton, Md.*

2384. *Nellie Hicks, Seneca Falls, N. Y.*

Correction to **2383.**

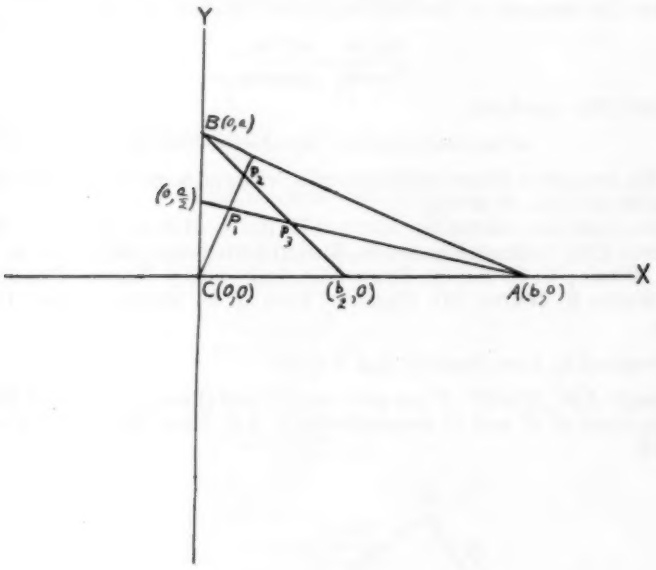
Leon Bankoff in his solution as sent to the printer states: "The locus of N is an equilateral hyperbola with asymptotes $x=-R$, $y=2R$." The published solution was in error.

A proof, this is, of poor proof reading!

2395. *Proposed by Richard H. Bates, Milford, N. Y.*

In right triangle ABC, with C the right angle, if medians are drawn to sides a and b, and an altitude to c, show that these three lines for an interior triangle whose area is given by

$$K=\frac{ab(b^2-a^2)^2}{6(a^2+c^2)(b^2+c^2)}.$$



Solution by J. H. Means, Huston-Tillotson College

Placing the triangle as shown, the equations of the two medians and altitude are

$$ax+2by=ab$$

$$2ax+\ by=ab$$

$$bx-\ ay=0$$

Solving these three equations simultaneously we get the vertices of the interior triangle:

$$P_1\left(\frac{a^2b}{b^2+c^2}, \frac{ab^2}{b^2+c^2}\right), \quad P_2\left(\frac{a^2b}{a^2+c^2}, \frac{ab^2}{a^2+c^2}\right), \quad P_3\left(\frac{b}{3}, \frac{a}{3}\right),$$

$$K=\frac{1}{2}\begin{vmatrix} \frac{a^2b}{b^2+c^2} & \frac{ab^2}{b^2+c^2} & 1 \\ \frac{b}{3} & \frac{a}{3} & 1 \\ \frac{a^2b}{a^2+c^2} & \frac{ab^2}{a^2+c^2} & 1 \end{vmatrix} = \frac{ab(a^2-b^2)^2}{6(a^2+b^2)(b^2+c^2)}.$$

Solutions were also offered by: W. J. Cherry, North Riverside, Ill.: V. C. Bailey, Evansville College, Evansville, Ind.; A. R. Haynes, Tacoma, Wash.; C. W. Trigg, Los Angeles City College; Celia Conley, Liverpool, N. Y.; W. R. Warne, Mt. Pleasant, Ia.; Herta T. Freitag, Roanoke, Va.; and the proposer.

2396. *Proposed by L. H. Lange, Valparaiso University.*

Show that the slope, m, of a bisector of an angle between two lines satisfies $m^2=1$ if the slopes of the two given lines are reciprocals."

Solution by L. H. Lange, Valparaiso University

Let m_1 and m_2 be the slopes of the two given lines. Then the slope m is such that, when the tangents of the half angles are equated, we have

$$\frac{m_2-m}{1+mm_2}=\frac{m-m_1}{1+mm_1}$$

which yields the quadratic

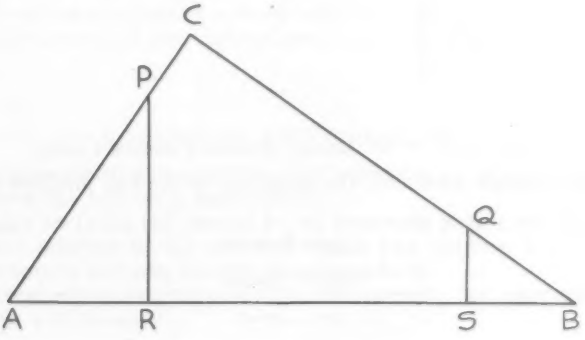
$$m^2(m_1+m_2)+m(2)(1-m_1m_2)-(m_1+m_2)=0.$$

Now, if the two given slopes are reciprocals, we have $m_1m_2=1$ and our expression for m yields $m=\pm 1$, or $m^2=1$.

Solutions were also offered by: Richard H. Bates, Milford, N. Y.; C. W. Trigg, Los Angeles City College; Charles H. Butler, Kalamazoo, Mich.; A. R. Haynes, Tacoma, Wash.; V. C. Bailey, Evansville, Ind.; Scoby Warne, Unionhill, New Jersey; Walter R. Warne, Mt. Pleasant, Iowa; J. H. Means, Austin, Texas and proposer.

2397. *Proposed by Leon Bankoff, Los Angeles.*

In triangle ABC, $C=90°$, P is a point on AC and Q is a point on CB. If R and S are projections of P and Q respectively on AB, show that $PR\cdot QS=AS\cdot BR-RS\cdot AB$.



Solution by Charles H. Butler, Kalamazoo, Michigan

From similar triangles it is clear that

$$\frac{PR}{AR}=\frac{SB}{QS}.$$

Therefore

$$\begin{aligned}PR\cdot QS&=AR\cdot SB\\&=(AB-BR)(AB-AS)\\&=\overline{AB}^2-AB\cdot AS-AB\cdot BR+BR\cdot AS\\&=BR\cdot AS+AB(AB-AS-BR)\\&=BR\cdot AS+AB(-RS)\\&=BR\cdot AS-AB\cdot RS.\end{aligned}$$

Solutions were also offered by: Leonard Antel, Detroit, Mich.; Brother Felix John, Philadelphia; Robert J. Orr, Cedar Falls, Iowa; Marian Craig, Felton, California; Hazel Holt, Willard, New York; Richard H. Bates, Milford, N. Y.; Robert Cawley, Dunmore, Pa.; W. J. Cherry, North Riverside, Ill.; Herta Taussig Freitag, Roanoke, Va.; Margaret Joseph, Milwaukee, Wis.; C. W. Trigg, Los Angeles City College; Douglas Kim, Silver Spring, Maryland; and proposer, also Robert Hartshorne, Exter, N. H.

2398. *Proposed by Roy Wild, University of Idaho.*

Use this as a definition:

$$\arctan x=\int_0^x\frac{dt}{1+t^2}.$$

Show from this that for $0\leqq x<1$

$$2\arctan x=\arctan\frac{2x}{1-x^2}.$$

Solution by H. M. Feldman, St. Louis, Mo.

Let

$$u=\frac{2t}{1-t^2},$$

then

$$du=\frac{2(1+t^2)dt}{(1-t^2)^2},$$

and

$$1+u^2=\frac{(1+t^2)^2}{(1-t^2)^2},$$

thus

$$\frac{du}{1+u^2}=\frac{2dt}{1+t^2}.$$

From which we get:

$$\int_0^x\frac{2dt}{1+t^2}=\int_0^{2x/(1-x^2)}\frac{du}{1+u^2},$$

or

$$2 \arctan x = \arctan \frac{2x}{1-x^2}.$$

Solutions were also offered by: Walter R. Warne, Mt. Pleasant, Iowa; Roy Wild, University of Idaho; Richard R. Williams, Jr., Marshall, Texas; C. W. Trigg, Los Angeles City College.

2399. *Proposed by Leon Bankoff, Los Angeles.*

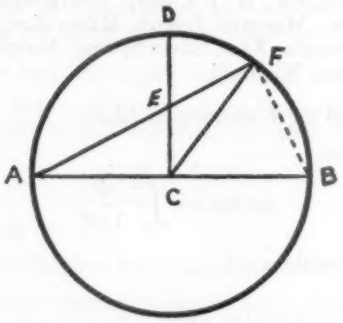
In a circle center C, AB is a diameter, and E is the mid-point of the radius DC perpendicular to AB. AE extended meets arc BD in F. Show by elementary geometry that the area of traingle ECF is equal to

$$3(AC)^2/20.$$

Solution by V. C. Bailey, Evansville College, Evansville, Ind.

Triangle ACE is similar to triangle AFB. Then

$$AF = \frac{4}{\sqrt{5}} r, \quad \text{and} \quad BF = \frac{2}{\sqrt{5}} r.$$



The area of triangle FCB equals $\frac{1}{2}$ the area of triangle ABF.

$$\triangle ECF = \triangle ABF - \triangle ACE - \triangle FCB$$

$$\triangle ECF = \frac{8}{5} r^2 - \frac{1}{4} r^2 - \frac{4}{5} r^2.$$

Then

$$\triangle ECF = \frac{3}{20} r^2 = \frac{3}{20} (AC)^2.$$

Solutions also offered by: William H. Hornbrook, Eaton, Ohio; Charles H. Butler, Kalamazoo, Mich.; David Rappaport, Chicago, Ill.; John Hoffman, Arcadia, Calif.; Norman Anning, Alhambra, Calif.; Brother Felix John, Philadelphia; Oren C. Hawse, Eloy, Arizona; A. R. Haynes, Tacoma, Wash.; Elizabeth Vreeland, Baileytown, N. Y.; Inez Purcell, Yorktown, Virginia; Richard H. Bates, Milford, N. Y.; Walter R. Warne, Mt. Pleasant, Iowa; W. J. Cherry, North Riverside, Ill.; Edward F. Sidor, Chicago; Joseph Kennedy, Greenfield, Indiana; C. W. Trigg, Los Angeles City College; V. C. Bailey, Evansville, Ind.; and proposer.

2400. *Proposed by C. W. Trigg, Los Angeles City College.*

There are two five digit numbers whose last three digits are unlike and in geometric progression not necessarily in natural order, and whose squares contain the ten digits once each. Find them and show that there are no others.

Solution by C. W. Trigg, Los Angeles City College

$1023456789 < N^2 < 9876543210$ so $31992 < N < 99381$ and $N \equiv 0 \pmod 3$. There are only three sets of unlike digits which are in G.P. Hence reference to a table of squares of numbers less than 1000 shows that N must terminate in 124, 139, 142, 193, 214, 241, 248, 319, 421, 428, 482, 824, 842, 913 or 931. Extending the examination to a table of squares for $N < 10{,}000$ and applying the criteria of divisibility by 3 reduces the eligible values to 232 numbers. Now since the first four digits of N^2 can be read from the same table, the appearance of duplicate digits further reduces the possible values to 19 numbers, namely 35214, 46248, 49824, 53319, 59931, 71193, 77124, 77241, 79428, 82428, 83241, 83421, 83913, 85428, 91248, 92214, 95124, 98241, and 98421 When these are squared, the only two squares with distinct digits are found to be

$$(92214)^2 = 8503421796$$

and

$$(91248)^2 = 8326197504.$$

It will be observed that in the latter value of N the last *four* digits are in G.P. and in natural order.

A solution was also offered by Richard H. Bates, Milford, N. Y.

LATE SOLUTIONS
HIGH SCHOOL HONOR ROLL

The Editor will be very happy to make special mention of high school classes, clubs, or individual students who offer solutions to problems submitted in this department. Teachers are urged to report to the Editor such solutions.

Editor's Note: For a time each high school contributor will receive a copy of the magazine in which the student's name appears.

For this issue the Honor Roll appears below.

2390, 1, 2. *John Hewett, Upper Canada College.*

2390, 1, 6, 7. *Nick Ross, Upper Canada College, Toronto.*

2395, 6, 7, 9. *Edgar Feedman, Brooklyn; Geoffrey Kandall, Far Rockaway, N. Y.*

2397. *Larry Jackson, Kipton, Ohio; Edmund Maske, Brooklyn; John Finnerty, Brooklyn; Thomas Maye, Brooklyn; Gerard McVicker, Richmond Hill, N. Y.*

2389. *Marian Craig, Felton, Calif.*

2396, 7, 9. *T. T. Gibson, Upper Canada College.*

2397, 9. *J. Hewett, Upper Canada College.*

PROBLEMS FOR SOLUTION

2419. *Proposed by Felix John, Philadelphia.*

In circle O, P is the mid-point of chord AB. Chords RS and TV pass thru P. RV cuts AP at M, and ST cuts PB at N. Prove, by high school geometry if possible, that $MP = PN$.

2420. *Proposed by C. W. Trigg, Los Angeles City College.*

Show that if the condition $a^2+b^2+c^2 = 2(ab+bc+ca)$ exists between the sides of a triangle, then its incircle and its three excircles can be arranged in a configuration such that each circle is tangent to the other three, not all at the same point.

2421. *Proposed by C. W. Trigg, Los Angeles City College.*

Solve the equation

$$\sum_{m=1}^{n} \cos mx = 1.$$

2422. *Proposed by Leon Bankoff, Los Angeles.*

"A realtor exchanged two square plots with integer sides and of different areas for one rectangular plot equal to the combined areas of the other two. He then subdivided the rectangular plot into eight square plots with integer sides, each of which was smaller than either of the two original square plots. What are the least possible dimensions of all the plots, using one yard as a unit?"

2423. *Proposed by Leon Bankoff, Los Angeles.*

Side BC of triangle ABC is divided by P so that $BP/PC=(BA+AP)/(AP+AC)$. Show that the incircles of triangle ABP and ACP are equal.

2424. *Proposed by Felix John, Philadelphia.*

Find the area of the triangle whose sides are

$$\frac{\sqrt{673}}{2}, \quad \frac{\sqrt{505}}{2} \quad \text{and} \quad 2\sqrt{37}.$$

BOOKS AND PAMPHLETS RECEIVED

DIFFERENTIAL EQUATIONS WITH APPLICATIONS, by Herman Betz, Paul B. Burcham, and George M. Ewing, *Department of Mathematics, University of Missouri, Columbia, Missouri.* Cloth. Pages x+310. 13.5×21 cm. 1954. Harper and Brothers, 49 East 33d Street, New York 16, N. Y. Price $4.50.

ANALYTIC GEOMETRY, Second Edition, by Edward S. Smith, M.E., Ph.D., *Professor of Mathematics, University of Cincinati;* Meyer Salkover, Ph.D., *Professor of Mathematics, University of Cincinnati;* and Howard K. Justice, C.E., Ph.D., *Assistant Dean, College of Engineering and Professor of Mathematics, University of Cincinnati.* Cloth. Pages xiii+306. 13.5×21 cm. 1954. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. Price $4.00.

ALGEBRA, Book Two, by C. A. Smith, *Chairman, Mathematics Department, Central High School, St. Louis, Missouri;* W. Fred Totten, *President, Flint Junior College, Flint, Michigan;* and Harl R. Douglass, *Director College of Education, University of Colorado, Boulder, Colorado.* Cloth. Pages ix+502. 13.5×20.5 cm. 1954. Row, Peterson and Company, 1911 Ridge Avenue, Evanston, Ill. Price $2.72.

MATTER ENERGY MECHANICS, by Dr. Jakob Mandelker, *Associate Professor of Mechanics, Georgia Institute of Technology, Atlanta, Georgia.* Cloth. Pages ix+73. 14×22 cm. 1954. Philosophical Library, Inc., 15 East 40th Street, New York 16, N. Y. Price $3.75.

HOW THE EARTH IS MADE, by Peter Hood. Cloth. 64 pages. 17×23 cm. 1954. Oxford University Press, 114 Fifth Avenue, New York 11, N. Y. Price $2.50.

PRINCIPLES OF BIOLOGY, by W. Gordon Whaley, Osmond P. Breland, Charles Heimsch, Austin Phelps, and Glenn S. Rabideau, *The University of Texas, Austin, Texas.* Cloth. Pages ix+694.16×23.5 cm. 1954. Harper and Brothers, 49 East 33d Street, New York 16, N. Y. Price $6.00.

SUPERJET, A SPY-RING THRILLER, by Lewis Zarem and Ray Cantwell. Cloth.

126 pages, 13×20.5 cm. 1954. E. P. Dutton and Company, Inc., 300 Fourth Avenue, New York 10, N. Y. Price $2.50.

MODERN CHEMISTRY, by the late Charles E. Dull, *West Side High School, Newark, New Jersey.* Rewritten and Revised by William O. Brooks, *Technical High School, Springfield, Massachusetts,* and H. Clark Metcalfe, *Shaler Township High School, Glenshaw, Pennsylvania.* Cloth. Pages xi+587. 15×23.5 cm. 1954. Henry Holt and Company, 383 Madison Avenue, New York 17, N. Y. Price $3.88.

SCIENCE—A STORY OF OBESRVATION AND EXPERIMENT, Book 1, by Ira C. Davis, *School of Education, University of Wisconsin, Madison, Wisconsin;* John Burnett, *Principal of the Arlington Memorial School, Arlington, Vermont;* and E. Wayne Gross, *Junior and Senior High School, University School, Bloomington, Indiana.* Cloth. Pages viii+341. 15×23.5 cm. 1954. Henry Holt and Company, 383 Madison Avenue, New York 17, N. Y. Price $3.16.

SCIENCE—A STORY OF EXPERIMENT AND DISCOVERY, Book 2, by Ira C. Davis, *School of Education, University of Wisconsin, Madison Wisconsin;* John Burnett, *Principal of the Arlington Memorial School, Arlington, Vermont;* and E. Wayne Gross, *Junior and Senior High School, University School, Bloomington, Indiana.* Cloth. Pages viii+438. 15×23.5 cm. 1954. Henry Holt and Company, 383 Madison Avenue, New York 17, N. Y. Price $3.28.

LIVING CRAFTS, by G. Bernard Hughes. Cloth. 192 pages. 12.5×20 cm. 1954. Philosophical Library, Inc., 15 East 40th Street, New York 16, N. Y. Price $4.75.

NEW SENIOR SCIENCE, by George L. Bush and Will S. Thompson, *Department of Chemistry and General Physical Science, Kent State University, Kent, Ohio.* Cloth. Pages ix+642. 15.5×23.5 cm. 1954. American Book Company, 55 Fifth Avenue, New York 3, N. Y. Price $4.28.

DIFFERENTIAL AND INTEGRAL CALCULUS, Fifth Edition, by Clyde E. Love, Ph.D., *Professor Emeritus of Mathematics in the University of Michigan,* and Earl D. Rainville, Ph. D., *Professor of Mathematics in the University of Michigan.* Cloth. Pages xiv+526. 15×23.5 cm. 1954. The Macmillan Company, 60 Fifth Avenue, New York 11, N. Y. Price $5.75.

MATHEMAGIC. MAGIC, PUZZLES AND GAMES WITH NUMBERS, by Royal Vale Heath. Paper. 126 pages. 13×20.5 cm. 1953. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. Price $1.00.

BLYTH SECOND-YEAR ALGEBRA TEST, by Isobel Blyth, *Michigan State College, East Lansing, Michigan.* $2.80 net per Package of 35. Machine-scored Answer Sheets $1.15 net per 35. Specimen set 35 cents postpaid. World Book Company, Yonkers-on-Hudson, N. Y.

Butter dish has a "built-in refrigerator" in its base which absorbs cold from the kitchen refrigerator while stored in the usual manner. On the table, this base keeps the butter cold through the entire meal, "even in the hottest temperature," the maker reports.

BOOK REVIEWS

THE SCIENTIFIC ADVENTURE, ESSAYS IN THE HISTORY AND PHILOSOPHY OF SCIENCE, by Herbert Dingle, *Professor of the History and Philosophy of Science, University College, London.* Cloth. Pages vii+372. 15×22.5 cm. 1953. Philosophical Library Inc., 15 East 40th St., New York 16, N. Y. Price $6.00.

As indicated in the title, the essays are assembled in two sections; the first presenting the history and the second the philosophical aspects of science.

The eight chapters in Part One deal principally with astronomy and physics giving one chapter each to Copernicus, Galileo and Thomas Wright with five that are less biographical. Of the eleven philosophical papers several challenge attention by their titles. In this list one would include: Science and the Unobservable, Physics and Experience, Science and Ethics and Science and Religion. The more conventional titles are Rational and Empirical Elements in Physics, Laws of Nature and Theory of Measurement. In this latter section the author strives, with much profit to the reader, to organize the regularities of experience into a consistent whole inter-related by natural laws but not closed to evidence which cannot yet be brought into this pattern.

The style is, at times, pedantic and heavy, the demands made upon the reader are more those of the philosopher than of the scientist. The diction is not likely to appeal to the casual reader who wishes to browse for relaxation. For the student who comes to the book with well defined queries, however, there may not always be an answer but very probably it will send him away with added zest for the search.

B. CLIFFORD HENDRICKS
Longview, Washington

SCIENCE—A STORY OF OBSERVATION AND EXPERIMENT, Book 1, and SCIENCE—A STORY OF EXPERIMENT AND DISCOVERY, Book 2, by Ira C. Davis, *School of Education, University of Wisconsin, Madison, Wisconsin;* John Burnett, *Principal of the Arlington Memorial School, Arlington, Vermont;* and E. Wayne Gross, *Junior and Senior High School, University School, Bloomington, Indiana.* Book 1, Cloth. Pages viii+341. Price $3.16. Book 2, Cloth. Pages viii+438. Price $3.28. 1954. Henry Holt and Company, 383 Madison Avenue, New York 17, N. Y.

These two books, with "Science—A Story of Discovery and Progress" reviewed in October 1952, make up the science series for junior high school. These books are highly illustrated, many of the photographs and diagrams appearing in color. The aim throughout was to select the science concepts suitable for the student at each grade level. As the formal study of science begins in the junior high school the more elementary topics are found in Book 1. This consists of ten units of study. Each unit is divided into small sections, each dealing with a particular problem. A preview gives the student the historical background of the unit and a full page picture for inspiration, stimulus for inquiry, and a basis for more intensive study for special reports. This is followed by a list of questions to direct the thought, and a list of the new words with pronunciations and definitions. Pupil activities suggesting experimental work, which can easily be done by the pupil at home or at school with a limited amount of apparatus, are given for each section. Short lists of review questions follow each section. At the close of each unit is a brief summary of the discussion, some questions for review, a list of topics for special reports, and a list of questions for stimulating thought. A set of very brief biographies of the scientist mentioned and a list of interesting books are given.

Book 2 consists of twelve units and follows the same general arrangement as in Book 1.

This series is very attractive and well arranged for study and for the conveni-

ence of the teacher, but as is usually the case with a first edition, it is easy to find fault with the presentation; e.g. many of the diagrams need further explanation and in some cases complete revision. It would be difficult for a timberman from the great northwest forests to see the "logs" by the fireplace on page 4 of Book 1. Just how does the prism disperse the white light to produce the spectrum on page 117? In Book 2, page 15, will the left pan of the balance be depressed when the air is in the balloon rather than outside? How does Fig. 4–5 on page 106 show "how a refracting telescope works?" In Fig. 5–3, page 138, how is the higher temperature shown? In Fig. 5–7, page 142, can the difference in weight between a cubic foot of dry air and a cubic foot of damp air be shown by using a balance of the type shown? These are some of the questions pupils will ask. They want to see how the experiments work. Some improvements should be made in an early revision.

G. W. W.

FIRST COURSE IN CALCULUS by Hollis R. Cooley, *Washington Square College, New York University*. Cloth. 15.5×23.5 cm. Pages xii+643. John Wiley and Sons, Inc., 440 Fourth Ave., New York 16, N. Y. 1954. Price $6.00.

One first impression received upon picking up this text is the large number of pages. The author states that this is not a mere exercise book—it is meant to be read. Certainly if the student will make the effort, he can read and understand much of this book without further explanation (the question as to whether he will make the effort is undecided). The text explanations are more complete than in many current texts, at the same time the standard of rigor equals if it does not exceed that of the general run of texts.

Certain features seem exceptionally good: a first chapter introduces the student to the subject of calculus, and tries to let him see what some of the major problems are. To many students, this will seem much less discouraging than the practice of starting at once on, for example, some theorems on limits (which to the student seem useless and uninteresting). In his treatment of limits, the author makes use of the delta-epsilon concept. The material on page 87 points out to the student that a curve may have a maximum point where the slope is not equal to zero, and includes the case where the curve has a maximum point at an end point. Differential equations are introduced early, and used thereafter at intervals, rather than being isolated in a single chapter. There is a well written chapter of some thirty pages on the material of solid analytic geometry. When the operation of inverse differentiation is introduced the notation $D^{-1}f(x)$ is used, which probably indicates the nature of the operation better than the integral sign. As the material develops, this notation of D^{-1} is gradually dropped.

One might ask whether the book has any bad features. It might better be stated that some aspects will mean that it is not suited to a particular course, or the ideas or wishes of a particular instructor or department. Some would prefer that some elementary integration be introduced earlier in the text; for example, the concept of moments does not appear until page 393. As another example of a situation where some instructors may prefer another order of topics, inflection points are discussed on page 74 and on page 89, but it is not until page 323 that there is any great use made of such points as applied to curve tracing. On page 3 one finds: "If the rule prescribes a value of y for each value of x, then y is said to be a function of x." This follows the statement that for a functional relation between x and y some rule must exist whereby a value of one of these variables can be found corresponding to each value of the other. Some will question the necessity for the existence of *a rule*. Probably this depends on what one defines as *a rule*.

In general the number of problems seems ample, though there are some exceptions, for example, page 82, page 156. Answers to odd numbered exercises appear in the back. Definitely this book should be considered when a new text is adopted.

CECIL B. READ
University of Wichita

THEORY OF EQUATIONS, by Cyrus Colton MacDuffee, *Professor of Mathematics, University of Wisconsin.* Cloth. Pages vii+120. 15½×23.5 cm. John Wiley and Sons, Inc., 440 Fourth Ave., New York 16, N. Y. 1954. Price $3.75.

This is a text book for a college course on the junior or senior level. The content of courses in theory of equations varies quite widely—this text is somewhat traditional, with particular emphasis on the theory of polynomials. One departure from tradition is the omission of material on matrices and determinants, the author feels this material might be a portion of a following course. A knowledge of the calculus is assumed in this text.

Some of the topics covered include (this is not a complete listing): partial fractions, linear systems in n unknowns, rational solutions and integral roots, Horner's and Newton's methods and the rule of false position (there is no discussion of methods of finding complex roots of higher degree equations), symmetric functions, resultants and discriminants, and introduction to the concept of fields and rings.

At some points the number of problems seems somewhat inadequate; instructors wishing problems from applied fields will find very few. Answers are provided to some but not all problems.

The length of the text is such that most if not all the material can be covered in a one semester course; certainly this text should not be omitted from consideration if one seeks a text for a course covering essentially the traditional material.

CECIL B. READ

AN ANALYTICAL CALCULUS FOR SCHOOL AND UNIVERSITY by E. A. Maxwell, *Fellow of Queens' College, Cambridge.* Volume I. Cloth. 14.5×22.5 cm. Pages xii+165. Cambridge University Press, 32 E. 57th St., New York 22, N. Y. 1954. Price $2.75.

This British text covers the basic ideas of differentiation and integration for the elementary algebraic and trigonometric functions, logarithmic and exponential functions are not treated. Approximately half the book is devoted to differentiation, then integration is the main topic for the rest of the text.

It is doubtful if the book would be a satisfactory text in our colleges, although it might be used in a special situation. The number of problems is in several cases very small; answers appear in the back of the book. As examples of places where the book tends to differ from our traditional texts, one might mention: the use of the epsilon notation with the first introduction of limits; the use of the small Greek letter delta rather than the capital letter, to indicate an increment; the term *gradient* rather than *slope;* the term differential coefficient rather than derivative; no use of the principal values of the inverse trigonometric functions, resulting in an ambiguous sign for the derivatives; introduction of the topic of differentiation of determinants. The reviewer did not like the choice of scale in the graphs of inverse trigonometric functions.

The book may be of value as a source of problems—there are many problems from examination boards of the British universities; it is likewise of interest to see what is being taught in another country.

CECIL B. READ

ELEMENTS OF STATISTICS by H. C. Fryer, *Professor of Mathematics; Statistician, Agricultural Experiment Station, Kansas State College.* Cloth. 15×23.5 cm. Pages viii+262. John Wiley and Sons, Inc., 440 Fourth Ave., New York 16, N. Y. 1954. Price $4.75.

When one looks for a text in a first course in statistics, it is obvious that the content of the course as well as the type of student are determining factors of the first magnitude. The publishers' literature would lead one to believe that this is an ideal text for a first course planned for students interested in various fields, without a heavy mathematical requirement. To this reviewer, it does not seem that this objective has been satisfactorily met. The concept of measures of cen-

tral tendency and of variability is covered in 34 pages, which many instructors will think inadequate. A statement on page 21 that the harmonic mean is used only in specialized circumstances leads one to wonder if certain other averages are always valid (Is it not true that *any* average is used only in specialized circumstances?). A problem on page 26 asked whether a certain distribution is *skewed*—a term not defined until page 32. In the second chapter references (2.11), (2.12) and (2.13) are used both for properties of averages and for formulas, which causes confusion. The author defines quartiles and deciles as subgroups, rather than points of division, a practice which has received rather caustic criticism from several authorities. The paragraph beginning on line 6 of page 7 implies that one could not trust companies to provide honest specimens for sampling; if this is correct certainly many purchasing agents should revise their procedures. Some of the graphs are poorly drawn, for examples see pages 93 and 157. On page 79 the author uses the American Experience Mortality Table; it would have required no more effort to utilize the CSO table now almost universally accepted by insurance companies.

Obviously opinions differ, and no one in search of a text should reject a book on the basis of a single reviewer's opinion. It would seem, however, that in some respects this book does not come up to the usual high standard of the publisher.

CECIL B. READ

CHEMISTRY FOR OUR TIMES, Second Edition, by Elbert C. Weaver, *Instructor, Phillips Academy, Andover, Massachusetts;* and Lawrence S. Foster, *Chief of the Technical Information Section, Watertown Arsenal Laboratory, Ordnance Corps, Department of the Army, Watertown, Massachusetts.* Cloth. Pages v +666. 16 × 24 cm. 1954. McGraw-Hill Book Co., Inc., 330 West 42nd Street, New York 36, N. Y. Price $4.12.

This is the second edition of a book that has been well received previously. In general, it follows the usual order of topics, as they are found in most high school chemistry texts.

The reviewer was pleased to observe that the authors began with principles that could be demonstrated in a high school chemistry laboratory rather than with the theory of sub-atomic structure which the student has to accept on faith. After all, chemistry is an experimental science and the reviewer feels that this text keeps this point of view throughout. It is not until pages 118 to 128 that the structure of the atom is presented and by this time quite a few chemical changes have been discussed. Unfortuntaely, the authors also relegated their presentation of chemical arithmetic to a later part of the book (pages 221 to 229). The reviewer feels that this should have been presented earlier in the course and could easily have been developed as a part of their Unit Three.

The subject matter is organized in ten attractive units, each of which has four to five chapters. Distributed within each chapter are excellent thought questions and review exercises. A good glossary of chemical terms is found in the appendix.

The publishing company has done a very good job. The paper is good, the print is easily read, and the double column format is used. Illustrations are ample and appropriate.

A high school teacher of chemistry using this book as a text should be able to teach practical chemistry to students who will never pursue the subject further and at the same time there is ample material for students who desire a background for college chemistry.

GERALD OSBORN
Western Michigan College
Kalamazoo, Michigan

THE WAY OF THE WORLD, by George H. T. Kimble, *Director of the American Geographical Society.* Pages x+123. Cloth. 1953. George Grady Press, New York, N. Y.

Here is a book for every Biologist, Geographer, Social Scientist and thinking layman who is seriously interested in the way of the world. Although a short

book by comparison, it has packed into it the deep understandings and revelations of the effects of Geography, Biology, culture and human nature upon man's world affairs, with an eye to the future.

It is a compilation of the 1952 Rushton Lectures and brings a different understanding to the study of geography. There is extensive covering of all subjects geographic, from the biology of the seas and deserts to the effects of climate on people. It cites agricultural changes, pictures early man in various parts of the earth in his attempt to establish his culture and discusses ecological problems like the importation of various animals and plants. Words are not minced. No reviews could do it justice.

This is a book of value to the secondary student and teacher, and should be available on every library shelf for the reading public.

JOHN D. WOOLEVER
Mumford High School
Detroit, Michigan

ASTROLOGY AND ALCHEMY: TWO FOSSIL SCIENCES, by Mark Graubard, *University of Minnesota*. Pages xi+382. 1953. Philosophical Library, Inc., New York, N. Y. Price $5.00.

The price of this book is a small amount indeed to pay for the results of the energy, research, and analysis of subjects so long neglected and misunderstood by educators and laymen alike. It is easy to say that Astrology and Alchemy were so far inferior to the scientific methods and practices of today, but this volume puts them in a new light, with a deep understanding of their value in view of the times they were popular.

The book is divided into two "books," the first devoted to Astrology, its importance in the development of Astronomy and the men responsible for the transition to the science as we know it today. Many diagrams are included to interpret the ideas prevalent during the various eras. Considering the equipment and facilities available, the various theories are quite plausible, if the reader projects himself back to the times in question. Their short-comings and reasons why they did not stand up, are also given.

The second half of the book traces the history of chemistry from centuries before Moses to the comparatively recent period where most chemistry texts start their history of chemistry.

A complete glossary is included as well as an extensive index which is unusual for this kind of book. This makes it of value for reference and research in school libraries.

Although primarily an historical work, it is deeply analytical, entertaining and offers much to those interested in the physical sciences and mathematics. Even though not recommended as light reading, it would be a valuable addition to any school or science library.

JOHN D. WOOLEVER

SOURCES OF FREE AND INEXPENSIVE TEACHING MATERIALS IN SCIENCE: CURRICULUM BULLETIN, by Hugh B. Wood and Howard Impecoven, *University of Oregon*. Paper. 9 pages. 1951. Edited by Hugh B. Wood, *Professor of Education*. Distributed by the School of Education, University of Oregon, Eugene, Ore. Price $.20.

This is one of many bulletins put out by The University of Oregon. Most educators realize the importance of such a list and have use for such a collection of sources. This particular list is mimeographed and is alphabetized according to the name of the company offering the materials. There are about two hundred different sources listed, with a key to their offerings, including pamphlets, maps, charts, graphs, posters, kits, models, pictures, slides, films, and film strips. Although the addresses are listed, the prices or a description of the materials relative to the specific subject covered is not included. It is primarily for intermedi-

ate and secondary schools, and was selectfully compiled and evaluated from a list of over four thousand firms and organizations.

JOHN D. WOOLEVER

PLASTIC MOUNTS FOR SCIENCE SPECIMENS; CURRICULUM BULLETIN, by Hiram M. Hunt. Paper. 16 pages. 1952. Edited by Hugh B. Wood, School of Education, University of Oregon, Eugene, Ore. Price $.25.

This is another handy mimeographed bulletin describing the general methods of making plastic mounts for science specimens, suitable for use by student or teacher. Very good details and directions are given. It lists materials, equipment, imbedding technique, curing and finishing. The directions are simple and easy to follow. Frequently directions are given to make simple equipment which otherwise is sold, by various plastic manufacturers.

A few diagrams are included, as well as a short bibliography.

JOHN D. WOOLEVER

INSECTS CLOSE UP, by Edward S. Ross. Paper. 1953. 80 pages. Published for The California Academy of Sciences by The University of California Press. Berkeley and Los Angeles, Cal. Price $1.50.

This paper bound booklet is an excellent collection of close up photographs of several dozen of our most common insects. Most are in black and white but a few color shots are included.

There is a short description of the insects' habits, life history, classification, and photographic data of each picture.

Several pages are devoted to insect collecting and nature photography which makes this publication a valuable guide for amateur entomologists and nature photographers. The science teacher might use this as a reference for his science shelf or use it as a source of pictures for opaque projector or bulletin board.

JOHN D. WOOLEVER

A TIME FOR SLEEP (HOW ANIMALS REST), by Millicent Selsam. Cloth. 56 pages. 1953. William R. Scott, Inc., New York. Price $2.00.

This profusely illustrated little book is a description of the sleeping habits of more than two dozen different animals, including, reptiles, birds, fish and mammals that are familiar to most elementary school children. It not only includes descriptions and methods of their sleeping habits but the illustrations show their exact positions while asleep. The latter is very important to children, it seems. Other items of interest are discussed in large type and simple easy to read sentences that would make this booklet one that can be kept on the childrens reading shelf for either use as reference material or leisure reading.

A distinction is made between sleeping and hibernation. This is rarely made in books written for young children. Each page is devoted to a separate animal and there is no plot, but the main theme of the book is interesting enough to hold the readers interest in the manner it is written. Although it is directed toward the juvenile reader, it might well be worthwhile for the Junior High school and high school science teacher to examine the contents of this book as it may answer many questions that arise in their classrooms, on this particular subject.

JOHN D. WOOLEVER

MISS PICKERELL GOES UNDERSEA, by Ellen MacGregor. 121 pages. 1953. Whittlesey House, McGraw-Hill Book Company, Inc., New York. Price $2.25.

This is the third in the series of Miss Pickerel science adventure stories written for the Elementary school reader. As with the others, the general plot of this adventure is light, easy to read and divided into the kind of chapter which permits the book to be read aloud by the teacher periodically, like a serialized story.

A ship sinks with one of Miss Pickerel's valuables aboard and she attempts to retrieve it by hiring a salvage company. After several minor complications, she

personally dives for her lost treasure with pleasant results for all.

All through this adventure are descriptions and discussions of the continental shelf, radar, sonar, snorkels, atomic-powered submarines, skin diving and under water television. This all lends a bit of scientific reality to an otherwise juvenile fantasy, and makes it entertaining reading for children from the second to the sixth grades.

JOHN D. WOOLEVER

FAST IS NOT A LADYBUG, by Miriam Schlein. 30 pages. 1953. William R. Scott, Inc. New York, N. Y. Price $1.75.

In the words of the author this is "A book about fast and slow things." In child-like drawings and a few sentences, this is an attempt to help a child understand the concept of slow and fast, or relative speeds.

Many of the examples used however, rocket ships, 389 hours, miles, etc. are most likely beyond the child's experience or understanding. If they did understand them, they would most certainly fully understand the difference between fast and slow, without reading about it in this manner.

Four sentences about a child planting a seed which grows into a rose, is used as an example of a "slow story," "because growing is always slow" and "because growing must be slow." The speeds of lightning, planes, trains, horses, etc. are compared, with the conclusion that some things are nice when slow, others when fast.

Although the author's intentions were good, a parent or teacher could probably do better with less expenditure of time, money and effort to get the same concept of relativity across to the child. With added footnotes, this book might might be a hit with the kindergarden crowd or maiden aunts seeking gifts for out of town pre-schoolers. Miss Schlein did far better with her book on "SHAPES."

JOHN D. WOOLEVER

FREE COURSES OF STUDY AVAILABLE TO TEACHERS AND SCHOOLS, prepared by Patrick Carr, *Director Curriculum Materials Center, Villa Grove Grade School, Villa Grove, Illinois.* 16 pages. 1953. Price $.50. Paper.

This staple bound, hectographed list is a directory of 186 guides, giving the titles, date of publication, number of pages and addresses where courses of study may be obtained on a variety of subjects. The list is alphabetized according to subject, from Art to Speech, and lists some general guides also.

There are more than a dozen that might be of interest to science teachers, particularly those interested in curriculum studies. The courses listed include those as of date of publication of the list, back to 1937, and represent almost every state of the union.

JOHN D. WOOLEVER

PLANE TRIGONOMETRY WITH TABLES, Second Edition. Donal H. Ballou, Ph.D., *Middlebury College.* Frederick H. Steen, Ph.D., *Allegheny College.* Ginn and Company. 1953. Cloth. vi+150 pp. Answers and tables. $3.25.

In Plane Trigonometry with Tables, 2nd Edition, the angle occupies the central position. The first chapter defines the general angle and its units of measurement —degree, radian, and mil. Rectangular coordinates are introduced. This is followed in the second chapter by the general-angle definitions of the six functions.

The work on identities is developed by examples in simplifying given expression. The student has the right member of the equation in the answer book. Here, the learner becomes familiar with fundamental relations and gains skill in the type of manipulation necessary in later work.

In this revision there is a chapter on complex numbers. The notation cis ϕ for $\cos\phi+i\sin\phi$ is used extensively in DeMoivere's formula. The new chapter on graphs goes as far as the general sine curve, simple harmonic motion, etc.

Chapter III, solution of right triangles, and Chapter VIII, solution of oblique triangles, are rich in problems of application from physics, surveying, navigation

and aviation. The use of logarithms is encouraged in both of these chapters. This topic is taught in a separate chapter.

A unique feature of the text is numerous sets of "Orals." These are a great help with new methods and formulas. They also provide material for drill and review.

FOREST MONTGOMERY
Lyons Township High School and Junior College
La Grange, Illinois

ELEMENTS OF CHEMISTRY, Revised Edition, by Raymond B. Brownlee, Robert W. Fuller, Jesse E. Whitsit, William J. Hancock, and Michael D. Sohon; *all have been successful teachers of chemistry in high schools of New York City.* Cloth. Pages xviii+708. 13.5 cm. by 21 cm. 1954. Allyn and Bacon, Inc., New York, N. Y.

The first impression one gains on examination of *Elements of Chemistry* is that it presents an attractive appearance. On the front cover is a kodachrome showing a feminine chemist surrounded by multicolored chemical apparatus, and on the back cover is another kodachrome, depicting sulfur mining by the Frasch method. Eight other kodachrome prints are scattered through the book. If the student reacts similarly to this reviewer, his first act will be to seek out these plates. What better motivation can a book provide than a glimpse at the colorful spectacle of industrial chemistry as portrayed by the "Blow" of a Bessemer converter and the mixing of paint pigments or the intriguing colors produced in the laboratory by the precipitation of certain sulfides? Diagrams are extensively used as visual aids.

Each chapter is a well integrated whole. This reviewer likes particularly the procedure of inserting "Self-Testing Exercises" at frequent intervals. These afford an opportunity for the student to check his understanding of preceding material before venturing unpreparedly into the next section; he tests his swimming ability before entering deep water. Each chapter has an adequate summary which completely defines all of the fundamental terms introduced in that chapter. At the end of each chapter are to be found the usual problems and questions. However, among these are application questions, starred for convenience, which provide a means for testing the individual student's ability to think with and apply the information learned.

The history of chemistry is deftly interwoven into the presentation and should serve to impress the student that chemistry is a vital science, still throbbing with growing pains. On page 177 is the interesting statement that Sir Humphrey Davy was knighted for his invention of the miner's safety lamp. In as much as Davy was knighted in 1812 and did not work on the safety lamp until about 1815—the lamp being put into use in mines in 1816—this reviewer is of the opinion that the authors have in mind the occasion of 1818 when Davy was made a baronet for his service to industry.

The order followed in developing a functional understanding of chemistry is in keeping with modern trends. Contrary to the prevailing tendency for chemistry books to have too high a reading level, this text seems to have a reading level closer to that of high school juniors and seniors. Nevertheless, this text like all chemistry texts, entitled to the name, has that one basic fault of which chemistry teachers tend to become incognizant; the student who desires to master the subject is faced with the almost insurmountable task of mastering, in one year, an extensive system of symbolization and a highly technical vocabulary. Perhaps this accounts for our low enrollments and high mortality rates in high school chemistry.

The text is organized in such a manner as to facilitate the adaption of the book to a course for the less academic students as well as to a course for those following a college-preparatory curriculum. Problems involving Boyle's and Charles' Laws and temperature scale conversions are treated in the appendix, making it possible for the teacher to introduce this material at his own discretion.

In the chapter entitled Principles of Chemical Reaction, the authors have deviated from the conventional terms for metathesis reactions, preferring to use the concept "New Pairing of Ions." The reviewer is not adverse to this change, for he believes that there are as many dissimilarities as similarities between single and double replacements. If this new terminology will eliminate some of the misinterpretations and confusion inherent in using similar names, the change will certainly be justified. Undoubtedly the phrase new pairing of ions will be much more descriptive of the actual mechanism of this type of reaction.

This book includes such timely topics as tracer isotopes, stratospheric rockets, jet propulsion, antibiotics, and recent nuclear discoveries. However, the basic fundamentals of chemistry have not been neglected for this reason; in fact the text has extended its coverage of basic chemistry to include some of the more recent theories. The book contains an excellent glossary and an adequate index.

As a whole this text consists of 708 excellently presented, up-to-date, accurate pages of chemistry. It certainly warrants careful consideration as a textbook if a change is contemplated.

JOSEPH E. DICKINSON
Watervliet, High School
Watervliet, Michigan

ELEMENTS OF PHYSICS, by D. Lee Baker, Raymond B. Brownlee, and Robert W. Fuller. Cloth. Pages ix+677. 15.5×23.5 cm. 1953. Allyn and Bacon, Inc., 2231 South Park Way, Chicago 16, Ill.

The high school physics instructor has the tremendous task of selecting a textbook which fits the needs of many kinds of students. In the average high school physics class today, a course of study must be designed which fits the needs of the college preparatory student, the general student, and the student who is terminating his study of science. The selection of the textbook is, perhaps, the most important phase in planning this varied course of study.

Elements of Physics attempts to meet these needs in various ways. The principles, laws, and terms are defined and presented in a brief form and in clear, easily readable language. The book emphasizes interest through the use of practical application and excellent photographs and sketches.

The organization is such that the student has an opportunity to review and evaluate his study throughout the course. Thought questions at various intervals in each chapter and discussion questions at the end of each chapter show the student whether he has mastered the facts, and also whether he can apply these facts in new situations.

Experiments which are designed to give the student training in the scientific method are placed throughout the book. These experiments, along with the problems which follow many chapters, bring out the importance of quantitative measurement and a real understanding of the principles studied.

The content includes units on matter and molecules, mechanics of fluids and solids, heat, sound, light, magnetism and electricity, electronics, transportation, and nuclear physics. This material is organized with first emphasis on the basic principles. This emphasis allows the teacher to bring in the "modern" in physics as he sees fit.

In the opinion of this reviewer, *Elements of Physics* is a very good text which should be applicable in any high school physics class.

ROBERT E. CARPENTER
Senior High School
Richmond, Indiana

Decorative compass for automobiles shows the motorist whether he is headed north, south, east or west. The easily read compass is floated in a small butyrate plastic globe with North and South America visible. At night, the markings are illuminated indirectly.